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String loop corrected hypermultiplet moduli spaces

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Abstract: Using constraints from supersymmetry and string perturbation theory, we determine the string loop corrections to the hypermultiplet moduli space of type II strings compactified on a generic Calabi-Yau threefold. The corresponding quaternion-Kähler manifolds are completely encoded in terms of a single function. The latter receives a one-loop correction and, using superspace techniques, we argue for the existence of a non-renormalization theorem excluding higher loop contributions.

Keywords: Supersymmetric Effective Theories, Superstring Vacua.

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Contents

1. Introduction

The couplings of the massless modes of type II string theory compactified on a Calabi-Yau threefold (CY_3) can be encoded in low energy effective actions (LEEA) with $\mathcal{N} = 2$ supersymmetry. These LEEA generally receive quantum corrections from the world sheet conformal field theory (α'-corrections) and from higher genus world sheets (g_s-corrections).

Perturbative LEEA are expanded in a double perturbation series in α' and g_s (see e.g. [1, 2] for a review). Both from a fundamental perspective, and in view of recent semi-realistic phenomenological applications to $\mathcal{N} = 1$ theories [3], it is important to determine the quantum structure of such LEEA. While the α'-corrections to the classical LEEA are well understood, finding the perturbative g_s corrections has remained an open problem.

As the main result of this paper we determine the (one-loop) g_s corrections to these LEEA. We find that the corrections are universal in the sense that they depend on the Euler characteristic of the CY_3 only. Furthermore, we argue in favor of a non-renormalization theorem which excludes higher loop contributions. We expect that these results also have implications in the context of Calabi-Yau orientifold compactifications where they could reveal new insights on the vacuum structure.

The starting point of our investigation are $\mathcal{N} = 2$, $d = 4$ supergravity actions [4] coupled to vector multiplets (VM) and hypermultiplets (HM) which provide the LEEA for type II strings compactified on a generic CY_3. Supersymmetry implies that the total moduli space $\mathcal{M}$ of these theories factorizes into a local product [5]

$$\mathcal{M} = \mathcal{M}_{\mathrm{VM}} \otimes \mathcal{M}_{\mathrm{HM}} \,, \tag{1.1}$$

where $\mathcal{M}_{\mathrm{VM}}$ and $\mathcal{M}_{\mathrm{HM}}$ are parameterized by the scalars of the VM and HM, respectively. Supersymmetry further dictates that $\mathcal{M}_{\mathrm{VM}}$ be a special Kähler (SK) manifold [6], and $\mathcal{M}_{\mathrm{HM}}$ a quaternion-Kähler (QK) manifold [7]. For compactifications of the type IIA string the volume modulus sits in a VM while the dilaton is in the HM sector.[1] This implies that $\mathcal{M}_{\mathrm{VM}}$ gets α'-corrections while $\mathcal{M}_{\mathrm{HM}}$ receives g_s-corrections only. Compactifying type IIB strings the volume modulus and the dilaton are both in the HM sector. Hence the HM sector receives both α' and g_s-corrections while the VM sector is classically exact.

The type IIB VM prepotential can be computed through knowledge of the Yukawa couplings (or triple intersection forms) for the given CY_3. Applying mirror symmetry this result can then be used to determine the VM couplings in the type IIA compactification including α'-corrections [8] (for a review, see [9]). This gives, at least in principle, the complete picture of the VM moduli space in these compactifications.

The corresponding picture in the HM sector is, however, less complete. This is mainly due to the lack of a (non-)perturbative duality that relates a classically exact sector of the M-theory moduli space to $\mathcal{M}_{\mathrm{HM}}$.[2] The classical result for $\mathcal{M}_{\mathrm{HM}}$ can be obtained by the (classical) c-map [10, 11] which relates the VM sector of the type IIA (IIB) to the HM sector of the type IIB (IIA) string compactification on the same CY_3. But beyond this classical result only little is known about string loop corrections to this sector. While we will elaborate on the perturbative corrections below, non-perturbative corrections due

[1] By this, we mean the four-dimensional dilaton in which the factorization property (1.1) holds. We will explain its relation to the string coupling constant in later sections.

[2] In principle, one could use the string-string duality between heterotic strings compactified on $T^2 \times K3$ and type IIA string theory on a $K3$-fibered CY_3 to obtain the fully quantum corrected result. Even though there are no g_s corrections to the HM moduli space on the heterotic side, this space remains poorly understood already at the classical level.

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to D-brane and NS5-brane instantons have been proposed in [14]. (See [15, 16] for some results about such instanton corrections to the universal hypermultiplet.)

In this paper we will determine the form of the one-loop corrections to the HM moduli space in a generic CY_3 compactification of type II strings. Instead of doing this by performing explicit string loop calculations, we will impose the constraints from $\mathcal{N} = 2$ supersymmetry, together with generic properties that string perturbation theory has to satisfy. Our starting point is the list of Strominger's [17] which summarizes the properties of the perturbatively corrected HM moduli spaces:

1. due to $\mathcal{N} = 2$ supersymmetry the quantum-corrected metrics should be quaternion-Kähler,
2. the corrections to the classical result should be subleading in the dilaton (g_s),
3. the Peccei-Quinn symmetries (cfg. eq. (2.5)) are preserved at the perturbative quantum level,
4. since string amplitudes with an odd number of RR fields vanish, the perturbations to the classical result always contain an even number of RR fields,
5. parity is a symmetry,
6. and the full perturbatively corrected metrics should be consistent with the known results from string loop computations [18, 19].

These conditions turn out to be sufficient to determine the HM metric, and our main result is given in eq. (4.17), together with (4.26) in type IIA, and (4.29) in type IIB.

In [19] these conditions have been implemented for the case of the universal hypermultiplet with the result that this sector receives non-trivial quantum corrections proportional to the Euler characteristic of the (rigid) CY_3. Subsequently, this result has been rewritten in superspace, in terms of a single function that determines all the components of the one-loop corrected moduli space metric [20]. The implementation of Strominger's list on QK metrics of arbitrary dimension has been considered in [21, 22], but remained inconclusive due to technical problems in enforcing the QK condition on the deformations of the classical HM moduli space.

In this paper we use the off-shell formulation of superconformal tensor multiplets [23, 24] to determine the perturbative corrections to $\mathcal{M}_{\mathrm{HM}}$. The main advantage of the off-shell formulation is that one can describe QK metrics, and therefore the effective action, in terms of a single function. Further simplifications occur when there are additional isometries, like the Peccei-Quinn symmetries present in string perturbation theory. For $4n$-dimensional QK metrics with $n + 1$ commuting shift symmetries one can use the duality between hyper- and tensor multiplets (TM) in four dimensions. In that case the off-shell description can be given in terms of $\mathcal{N} = 2$ tensor multiplets and is also known [25] to be determined by a single function which we will call H in the following.

At the classical level H has recently been constructed in [26], see also [27, 28] for related results. This function was found by describing the c-map [10, 11] off-shell. We here search for deformations of this map which satisfy the conditions (1) - (6) stated above, and find the general solution. The deformed functions H thus provide a quantum c-map which determine the perturbative corrections to the QK metrics arising from generic CY_3 compactifications of the type II string.

The rest of the paper is organized as follows. In section 2 we review the facts about supergravity theories and the classical c-map which are relevant in our construction. Section 3 outlines the superconformal quotient for the superconformal TM lagrangian which is then applied to the classical c-map. In section 4 the one-loop corrections and the resulting HM moduli spaces are constructed and in section 5 we argue for a non-renormalization theorem excluding higher loop corrections. Section 6 contains some discussion and an outlook. The technical details of our constructions are collected in appendix A.

2. Effective supergravity actions

In this section we describe the four-dimensional $\mathcal{N} = 2$ supergravity actions that provide the LEEA for type II strings compactified on a generic CY_3. We start be reviewing their tree level moduli spaces together with the c-map in Subsection 2.1. Subsection 2.2 discusses the off-shell formulation of Poincaré supergravity based on the superconformal calculus which will play a central role in our construction.

2.1 Tree level effective actions

Type II string compactifications on CY_3 yield four-dimensional $\mathcal{N} = 2$ supergravity theories including n_V vector and n_H hypermultiplets (or, equivalently, their tensor multiplet duals, see below). Denoting the Hodge numbers of the CY_3 by $h^{1,1}$ and $h^{2,1}$, compactifications of type IIA strings yield $n_V = h^{1,1}$, $n_H = h^{2,1} + 1$. In the type IIB case the Hodge numbers are interchanged, i.e., $n_V = h^{2,1}$, $n_H = h^{1,1} + 1$. $\mathcal{N} = 2$ supersymmetry further requires that the scalar manifolds of the theory factorize according to (1.1).

The VM scalars parameterize the manifold $\mathcal{M}_{\mathrm{VM}}$ which is local (projective) special Kähler [6, 5]. These manifolds are characterized by a so-called prepotential, a holomorphic function $F(X^\Lambda)$, $\Lambda = 1, \ldots, n_V+1$, which is homogeneous of degree two in X^Λ. Introducing projective coordinates

$$z^\Lambda = \frac{X^\Lambda}{X^1} = \{1, z^a\} \,, \quad a = 1, \ldots, n_V \,, \tag{2.1}$$

the metric and Kähler potential of $\mathcal{M}_{\mathrm{VM}}$ are given by

$$\mathcal{G}_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K} \,, \qquad \mathcal{K}(z, \bar{z}) = \ln \left(z^\Lambda N_{\Lambda\Sigma} \bar{z}^\Sigma\right) , \tag{2.2}$$

in which $N_{\Lambda\Sigma} = i \left(F_{\Lambda\Sigma} - \bar{F}_{\Lambda\Sigma}\right)$ and $F_\Lambda(X) = \frac{\partial}{\partial X^\Lambda} F(X)$ etc. Furthermore, the kinetic terms of the VM gauge fields are determined by the matrix

$$\mathcal{N}_{\Lambda\Sigma} = -i \bar{F}_{\Lambda\Sigma} - \frac{(Nz)_\Lambda (Nz)_\Sigma}{(zNz)} \,, \tag{2.3}$$

where $(Nz)_\Lambda = N_{\Lambda\Sigma} z^\Sigma$ and $(zNz) = z^\Lambda N_{\Lambda\Sigma} z^\Sigma$. When considering CY_3 compactifications the classical part of $F(X^\Lambda)$ is determined by the triple-intersection numbers of the CY_3. In

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compactifications of the type IIA string the prepotential additionally receives perturbative and non-perturbative α' corrections.

The HM scalars parameterize the manifold $\mathcal{M}_{\rm HM}$ which must be quaternion-Kähler [7]. At tree level in the string coupling constant, the corresponding HM lagrangians for the type IIA (IIB) compactification are related to the special Kähler geometry of the IIB (IIA) compactification on the same CY_3 via the c-map [10, 11].[3] Alternatively, refs. [29, 30] derived these lagrangians from a classical compactification of ten-dimensional IIA or IIB supergravity on a generic CY_3. The bosonic part of the resulting hypermultiplet lagrangians can then be written as (in conventions with $\kappa^{-2}=2$)

$$\begin{aligned} e^{-1}\mathcal{L} = & -R - \frac{1}{2}(\partial_\mu\phi)^2 + 2\mathcal{G}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \frac{1}{2}\mathrm{e}^{-\phi}\left(\mathcal{N}+\mathcal{N}\right)^{\Lambda\Sigma}\left|2\mathcal{N}_{\Lambda\Pi}\partial_\mu A^\Pi + i\partial_\mu B_\Lambda\right|^2 \\ & - \frac{1}{2}\mathrm{e}^{-2\phi}\left(\partial_\mu\sigma - \frac{1}{2}\left(A^\Lambda\partial_\mu B_\Lambda - B_\Lambda\partial_\mu A^\Lambda\right)\right)^2 . \end{aligned} \tag{2.4}$$

Here ϕ is the four-dimensional dilaton, σ is the dual scalar arising from the NS two-form, z^a, $a=1,\ldots,n_H-1$, are the geometric moduli (i.e., complex structure or Kähler moduli in IIA or IIB, respectively) and the $2n_H$ additional real scalars A^Λ, B_Λ arise from compactifying the (ten-dimensional) RR fields. Furthermore, $\mathcal{G}_{a\bar{b}}$ and $\mathcal{N}_{\Lambda\Sigma}$ are the metric (2.2) and gauge kinetic matrix (2.3) of the dual special Kähler geometry, and $(\mathcal{N}+\bar{\mathcal{N}})^{\Lambda\Sigma}$ is the inverse of $(\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma}$. Note that (2.4) is completely fixed by the underlying prepotential $F(X^\Lambda)$. In particular, this allows to determine the string tree-level α'-corrections to the type IIB hypermultiplet geometry by substituting the α'-corrected VM prepotential of the IIA compactification.

The compactification of the ten-dimensional tensor fields naturally induces certain symmetries in the resulting LEEA which have been studied in detail in ref. [31]. For our purpose it suffices to note that the lagrangian (2.4) is invariant under the $2n_H+1$ Peccei-Quinn symmetries

$$\delta\sigma = \epsilon + \frac{1}{2}\left(\alpha^\Lambda B_\Lambda - \beta_\Lambda\, A^\Lambda\right) , \quad \delta A^\Lambda = \alpha^\Lambda , \quad \delta B_\Lambda = \beta_\Lambda , \tag{2.5}$$

where ϵ, α^Λ and β_Λ are $2n_H+1$ real parameters. These isometries constitute a $2n_H+1$-dimensional Heisenberg algebra. Since these isometries originate from tensor fields in ten dimensions, this algebra is expected to be preserved at the perturbative quantum level [17].

These shift symmetries suggest that there should be a description of the lagrangian (2.4) in terms of tensor multiplets. In fact such a description naturally arises in compactifications of type II strings. For type IIA, one obtains $h^{2,1}$ hypermultiplets and one tensor multiplet. The latter can be dualized into a scalar yielding hypermultiplets only. Compactifying type IIB strings yields a double-tensor multiplet [33] and $h^{1,1}$ tensor multiplets. The bosonic part of the lagrangian for this system was found in [30], and reads

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm TM}_{\rm cl} = & -\frac{1}{2}(\partial_\mu\phi)^2 + 2\,\mathcal{G}_{a\bar{b}}\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}} + \frac{1}{2}\mathrm{e}^{-\phi}(\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma}\partial_\mu A^\Lambda\partial^\mu A^\Sigma \\ & + 2\,\mathcal{T}^{\rm cl}_{IJ}\,E^I_\mu E^{J\mu} + i(\mathcal{N}-\bar{\mathcal{N}})_{\Lambda\Sigma}\left[(\partial_\mu A^\Lambda)E^{\Sigma\mu} - 2(\partial_\mu A^\Lambda)A^\Sigma E^{0\mu}\right] . \end{aligned} \tag{2.6}$$

[3] Without gravity, one can also perform a (rigid) c-map, that maps vector multiplets to hypermultiplets. In terms of geometries, the map is between rigid special Kähler spaces and hyperkähler spaces [10, 12, 13].

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Here [4] $E^\mu = \frac{i}{2} e^{-1} \varepsilon^{\mu\nu\rho\sigma} E_{\rho\sigma}$ is the field strength of the antisymmetric tensor field $E_{\mu\nu}$. The index I runs over one more value than Λ, so $I = \{0, \Lambda\}$. This is because, compared to (2.4), both B_I and σ have been exchanged for tensors. The matrix $\mathcal{T}^{\rm cl}_{IJ}$ appearing in the tensor field kinetic term is given by

$$\mathcal{T}^{\rm cl}_{IJ} = {\rm e}^{\phi} \begin{bmatrix} {\rm e}^{\phi} - (\mathcal{N} + \bar{\mathcal{N}})_{\Lambda\Sigma} A^\Lambda A^\Sigma & \frac{1}{2}(\mathcal{N} + \bar{\mathcal{N}})_{\Lambda\Sigma} A^\Lambda \\ \frac{1}{2}(\mathcal{N} + \bar{\mathcal{N}})_{\Lambda\Sigma} A^\Sigma & -\frac{1}{4}(\mathcal{N} + \bar{\mathcal{N}})_{\Lambda\Sigma} \end{bmatrix} . \tag{2.7}$$

Dualizing the tensor field strengths back into scalars, one obtains the hypermultiplet lagrangian (2.4). This will be done explicitly in Subsection 3.2.

2.2 Off-shell formulation

As will become clear in this paper, working in an off-shell formalism turns out to be advantageous. The main reason is that the matter sectors in (2.4) and (2.6) can be elegantly recast into superspace language. Having such a formulation at hand considerably simplifies addressing the question of the loop corrections in the following.

Instead of considering the standard Poincaré supergravity described above, this off-shell formulation utilizes a gauge equivalent formulation based on the superconformal group. The multiplet containing the gravitational degrees of freedom in this locally superconformal invariant theory is called the Weyl multiplet. It contains the graviton and gravitinos as well as gauge fields for the U(1) and SU(2) R-symmetry groups that belong to the bosonic part of the superconformal group. Moreover, the theory can include any number of vector, hyper-, and tensor multiplets whose superconformal couplings have been worked out in [34, 5, 35] and [23]. In order to gauge fix to Poincaré supergravity one needs at least one vector and one hypermultiplet, which can act as compensators for the extra symmetries of the theory. Alternatively, as we will use later, the hypermultiplet compensators can be replaced by four compensating scalars in two tensor multiplets. Eliminating the auxiliary U(1) and SU(2) gauge fields, combined with appropriate gauge fixing conditions yields the Poincaré theory in which the moduli spaces $\mathcal{M}_{\rm VM}$ and $\mathcal{M}_{\rm HM}$ or $\mathcal{M}_{\rm TM}$ appear. This is the basic idea of the $\mathcal{N} = 2$ superconformal calculus [36] and the gauge fixing procedure is called the superconformal quotient.

The scalars of the vector, hyper- and tensor multiplets featuring in the superconformal theory parameterize the scalar manifolds $\mathcal{M}^{\rm SC}_{\rm VM}$, $\mathcal{M}^{\rm SC}_{\rm HM}$ and $\mathcal{M}^{\rm SC}_{\rm TM}$, respectively. The superscript "SC" indicates that the corresponding manifolds characterize a superconformal theory. The relations between these manifolds and their counterparts in Poincaré supergravity are summarized in figure 1. It turns out that $\mathcal{M}^{\rm SC}_{\rm VM}$ is a rigid (affine) special Kähler manifold of real dimension $2n_V + 2$. Its metric is completely determined by the holomorphic prepotential $F(X^\Lambda)$ homogeneous of second degree. This prepotential defines the Kähler potential K and the metric $N_{\Lambda\Sigma}$ on $\mathcal{M}^{\rm SC}_{\rm VM}$ as

$$K = i\left(\bar{X}^\Lambda F_\Lambda(X) - X^\Lambda \bar{F}_\Lambda(\bar{X})\right) , \qquad N_{\Lambda\Sigma} = i\left(F_{\Lambda\Sigma} - \bar{F}_{\Lambda\Sigma}\right) . \tag{2.8}$$

[4] We use Pauli-Källén conventions where $\varepsilon^{0123} = i$, so E^μ is real.

$$
\begin{tikzcd}
\mathcal{M}^{\rm SC}_{\rm TM} \arrow[rr, leftrightarrow, "\text{scalar-tensor duality}"] \arrow[d, leftrightarrow] & & \mathcal{M}^{\rm SC}_{\rm HM} \arrow[d, leftrightarrow] & \otimes & \mathcal{M}^{\rm SC}_{\rm VM} \arrow[d, leftrightarrow, "\text{superconformal quotient}"] \\
\mathcal{M}_{\rm TM} \arrow[rr, leftrightarrow, "\text{scalar-tensor duality}"'] & & \mathcal{M}_{\rm HM} & \otimes & \mathcal{M}_{\rm VM}
\end{tikzcd}
$$

Figure 1: Relations between the scalar manifolds featuring in the conformal (top) and Poincaré supergravity (bottom). The vertical arrows indicate that these theories are related by the superconformal quotient while the horizontal arrows imply that, provided the HM scalar manifolds have suitable isometries, scalars are dual to antisymmetric tensor fields thus relating the corresponding hyper- and tensor multiplet scalar manifolds.

Taking the superconformal quotient of $\mathcal{M}^{\rm SC}_{\rm VM}$ then leads to the local special Kähler manifold $\mathcal{M}_{\rm VM}$ of real dimension $2n_V$ detailed above. Furthermore, while $\mathcal{M}_{\rm HM}$ is quaternion-Kähler, it turns out that its superconformal counterpart $\mathcal{M}^{\rm SC}_{\rm HM}$ is a hyper-Kähler cone whose geometrical properties are completely specified by a single function, the hyper-Kähler potential [37, 38]. The relation between hyper-Kähler cones and quaternion-Kähler manifolds was studied in more detail in [24, 39 – 41]. Furthermore, this map was utilized to construct LEEA for CY_3 compactifications undergoing flop and conifold transitions in [42].

For the purpose of this paper, the most convenient starting point is the superconformal TM lagrangian [23]. Building on earlier work [25], it turns out that the corresponding scalar geometry $\mathcal{M}^{\rm SC}_{\rm TM}$ is encoded by a single function $\mathcal{F}(x, v, \bar{v})$, which completely determines the lagrangian. This function can be expressed in terms of a contour integral,

$$
\mathcal{F}(v^I, \bar{v}^I, x^I) = -\mathrm{Im}\left[\frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{\mathrm{d}\zeta}{\zeta} H(\eta^I)\right] \, . \tag{2.9}
$$

Here $I, J = 0, \ldots, n_H + 1$ enumerates the tensor multiplets and the three scalars of each tensor multiplet appear in the combination

$$
\eta^I = \frac{v^I}{\zeta} + x^I - \zeta \bar{v}^I \, . \tag{2.10}
$$

The contour integral representation guarantees that the function $\mathcal{F}$ satisfies the following differential equation [43, 25]

$$
\mathcal{F}_{x^I x^J} + \mathcal{F}_{v^I \bar{v}^J} = 0 \, . \tag{2.11}
$$

Conformal invariance requires the function H to be homogeneous of degree one[5] under rescalings of η^I and have no explicit ζ dependence while $\mathcal{C}$ is a curve in the complex ζ plane. All this naturally follows from $\mathcal{N} = 2$ projective superspace [43, 44], in which

[5]This homogeneity has to be understood under the contour integral. Linear terms of the form $H \propto \eta$ vanish in the final lagrangian, while terms of the form $H \propto \eta \ln \eta$ are non-vanishing, but only homogenous of degree one up to terms that vanish in the final lagrangian. For more details, see e.g. [24].

the the v^I are the scalars coming from a $\mathcal{N}=1$ chiral superfield while the x^I are the real scalar fields of an $\mathcal{N}=1$ tensor multiplet. Together they compose to an $\mathcal{N}=2$ tensor multiplet. The tensor multiplet sector of the conformal supergravity theory is then completely specified by the function $H(\eta)$ which, besides being homogeneous of degree one, does not need to satisfy any further constraints. The rigidly superconformal tensor multiplet lagrangian is described by integrating the function $\mathcal{F}$ over the $\mathcal{N}=2$ projective superspace measure. The coupling to the Weyl multiplet is described in the next section.

The question then arises what is the function H that, upon taking the superconformal quotient, gives rise to the lagrangians (2.4) or (2.6). This was recently answered in [26], where it was shown that

$$H^{\rm cl}(\eta) = \frac{F(\eta^\Lambda)}{\eta^0}\,. \tag{2.12}$$

Here $F(X)$ is the holomorphic prepotential determining the VM sector of the dual type II compactification, but now evaluated as a function of the TM fields η^Λ while η^0 is an additional compensator. To establish that (2.12) indeed gives rise to the hypermultiplet lagrangian (2.4), it is convenient to work in a partially gauge-fixed version where $v^0 = \bar{v}^0 = 0$. Then the contour $\mathcal{C}$ can be taken around the pole $\zeta = 0$ in the complex ζ-plane. Assuming that $F(\zeta\eta^\Lambda)$ has no poles at $\zeta = 0$, the contour integral yielding the function $\mathcal{F}(v, \bar{v}, x)$ can then easily be evaluated. By dualizing the resulting conformal tensor multiplet theory to hypermultiplets and subsequently performing the superconformal quotient on the resulting hyper-Kähler cone $\mathcal{M}_{\rm HM}^{\rm SC}$ it was verified that the resulting metrics on $\mathcal{M}_{\rm HM}$ are indeed given by (2.4) [26]. We rederive this result in the next section.

Complementary to the classical result (2.12), ref. [20] constructed the function $H(\eta)$ encoding the one-loop corrected universal hypermultiplet lagrangian found by Antoniadis et. al. [19]. With $F(\eta) = -i(\eta^1)^2$ describing the classical universal hypermultiplet, the one-loop corrected metric can be obtained from

$$H^{\rm UHM}(\eta) = -i\frac{(\eta^1)^2}{\eta^0} + 4\,i\,c\,\eta^0\,\ln(\eta^0)\,, \tag{2.13}$$

where c is an a priori undetermined constant. In the gauge $v^0 = \bar{v}^0 = 0$ the contour $\mathcal{C}$ is taken around the origin. Alternatively the contour integral can be evaluated without making this partial gauge choice by choosing a different contour [20].

That the second term describes a one-loop term of order g_s^2 higher than the classical term can be understood as follows. The string coupling is a dimensionless quantity. The tensor multiplets have scaling dimensions, so only a ratio can be proportional to g_s. From the explicit calculation in [20], it follows that η^1/η^0 scales like g_s^{-1}. It is then easy to see that the second term is of order g_s^2 higher.

It should be clear then that the problem of how to incorporate string loop corrections to the lagrangians (2.4) and (2.6) is most easily done in the off-shell description. Combined with (2.12), it will turn out to be easy to generalize (2.13) to the case of more hypermultiplets. We discuss this in section 4.

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3. Tensor multiplet lagrangians

Before we discuss the loop corrections, we first construct the most general $\mathcal{N}=2$ supergravity action coupled to tensor multiplets in components [23]. This will enable us to compare with known results for string loop amplitudes to the effective action, which we discuss in the next section.

The natural starting point for our investigation is the superconformal tensor multiplet lagrangian [23] including n_H+1 tensor multiplets. This was also the starting point for constructing the classical conformal c-map [26] where upon determining the function $\mathcal{F}(v,\bar{v},x)$, the hyper-Kähler potential of the corresponding hypermultiplet geometry $\mathcal{M}^{\rm SC}_{\rm HM}$ was found as the Legendre transform of $\mathcal{F}(v,\bar{v},x)$ with respect to x^I. Subsequently the superconformal quotient was taken on the hypermultiplet side along the lines of [24].

Including a logarithmic correction of the form (2.13), however, this strategy faces the obstacle that the Legendre transform of $\mathcal{F}(v,\bar{v},x)$ with respect to x^0 involves solving a transcendental equation which cannot be done explicitly. To avoid this complication we take a different route through figure 1 and first take the superconformal quotient on the tensor side before dualizing the tensors to scalar fields. The superconformal quotient for TM is then subject of Subsection 3.1. In Subsection 3.2 we utilize this formalism to derive the classical hypermultiplet lagrangian (2.4) starting from eq. (2.12).

3.1 The superconformal quotient for tensor multiplets

We start by considering n_H+1 tensor multiplets which are conformally coupled to the Weyl multiplet. The bosonic degrees of freedom of the $\mathcal{N}=2$ tensor multiplet [32] consist of an antisymmetric tensor field $E_{\mu\nu}$ with field strength $E^\mu := \frac{i}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\partial_\nu E_{\rho\sigma}$, an SU(2) triplet of scalars $L^{ji}=L^{ij}=(L_{ij})^*$, $i,j=1,2$, satisfying the reality condition $L^{kl}=\varepsilon^{ki}\varepsilon^{lj}\,L_{ij}$, and a complex auxiliary field G which will play no role in the following. The bosonic part of the Weyl multiplet contains the vielbein $e_\mu{}^a$, an auxiliary field D, and the (non-dynamical) gauge fields $\vec{\mathcal{V}}_\mu$, A_μ, b_μ gauging the SU(2), U(1) and dilatations of the superconformal algebra. Furthermore, we have a dependent gauge field $f_\mu{}^\mu = \frac{1}{6}R - D$ which is related to special conformal transformations.

In order to make contact with the tensor multiplet geometry outlined in Subsection 2.1 we decompose the L^{ijI} as

$$L^{12\,I} \equiv \frac{1}{2} i\, x^I\,, \qquad L^{11\,I} \equiv v^I\,, \qquad L^{22\,I} \equiv \bar{v}^I\,. \tag{3.1}$$

In these coordinates the bosonic part of the superconformal tensor multiplet lagrangian [23] can be concisely written as

$$\begin{aligned} e^{-1}\mathcal{L} = {}& \mathcal{F}_{x^Ix^J}\left(\partial_\mu v^I\partial^\mu\bar{v}^J + \frac{1}{4}\partial_\mu x^I\partial^\mu x^J - E^I_\mu\, E^{J\mu}\right) + i\, E^I_\mu\left(\mathcal{F}_{v^Ix^J}\,\partial^\mu v^J - \mathcal{F}_{\bar{v}^Ix^J}\partial^\mu\bar{v}^J\right) \\ & + \frac{1}{2}\left(\vec{\mathcal{V}}_\mu\right)^{\rm T} M\,\vec{\mathcal{V}}^\mu + \vec{\mathcal{V}}_\mu\cdot\left(\vec{S}^\mu + \vec{T}^\mu\right) - 2\mathcal{F}_{x^Ix^J}\left(v^I\bar{v}^J + \frac{1}{4}x^Ix^J\right)\left(\frac{1}{3}R + D\right). \end{aligned} \tag{3.2}$$

The SU(2) triplet of gauge fields $\vec{\mathcal{V}}_\mu$ couples to the SU(2) currents

$$\vec{S}_\mu = -\frac{i}{2}\mathcal{F}_{x^I x^J}\begin{bmatrix} v^I\,\partial_\mu x^J - x^I\,\partial_\mu v^J \\ -\bar{v}^I\,\partial_\mu x^J + x^I\,\partial_\mu \bar{v}^J \\ 2\left(\bar{v}^I\,\partial_\mu v^J - v^I\,\partial_\mu \bar{v}^J\right)\end{bmatrix}\,, \tag{3.3}$$

and

$$\vec{T}_\mu = \mathcal{F}_{x^I x^J} E^I_\mu \left[v^J\,,\,\bar{v}^J\,,\,x^J\right]^{\mathrm{T}}\,, \tag{3.4}$$

containing derivatives of the scalar and tensor fields, respectively. Furthermore, the matrix M which appears in the term quadratic in $\vec{\mathcal{V}}_\mu$ is, in the canonical complex basis $\vec{\mathcal{V}}_\mu = (\mathcal{V}^+_\mu, \mathcal{V}^-_\mu, \mathcal{V}^3_\mu)$ given by

$$M = \begin{bmatrix} -\frac{1}{2}\mathcal{F}_{x^I x^J} v^I v^J & \frac{1}{2}\mathcal{F}_{x^I x^J}\left(v^I\bar{v}^J + \frac{1}{2}x^I x^J\right) & -\frac{1}{2}\mathcal{F}_{x^I x^J} x^I v^J \\ \frac{1}{2}\mathcal{F}_{x^I x^J}\left(v^I\bar{v}^J + \frac{1}{2}x^I x^J\right) & -\frac{1}{2}\mathcal{F}_{x^I x^J}\bar{v}^I\bar{v}^J & -\frac{1}{2}\mathcal{F}_{x^I x^J} x^I\bar{v}^J \\ -\frac{1}{2}\mathcal{F}_{x^I x^J} x^I v^J & -\frac{1}{2}\mathcal{F}_{x^I x^J} x^I\bar{v}^J & 2\mathcal{F}_{x^I x^J}\bar{v}^I v^J \end{bmatrix}\,. \tag{3.5}$$

Observe that the lagrangian (3.2) and in particular the metric on $\mathcal{M}^{\mathrm{SC}}_{\mathrm{TM}}$ is completely fixed by specifying the function $\mathcal{F}(v,\bar{v},x)$ which is subject to the conditions (2.11).

The superconformal quotient is performed by making a gauge choice for the SU(2) symmetry and dilatations together with eliminating the gauge fields $\vec{\mathcal{V}}_\mu$ and the auxiliary field D by their equations of motion. For the fields $\vec{\mathcal{V}}_\mu$ this is straightforward. Here we can use the freedom of performing SU(2) rotations to fix

$$v^0 = 0\,, \quad v^1 = \bar{v}^1\,, \tag{3.6}$$

and then eliminate the non-dynamical fields $\vec{\mathcal{V}}_\mu$.

The consistent elimination of D is slightly more complicated and requires introducing a conformal vector multiplet which provides the compensator for the U(1) symmetry acting in the vector multiplet sector. The relevant piece coming from the conformal vector multiplet lagrangian can readily be taken from [23] and reads[6]

$$e^{-1}\mathcal{L}^{\mathrm{VM}} = -i\left(F_\Lambda \bar{X}^\Lambda - \bar{F}_\Lambda X^\Lambda\right)\left(-\frac{1}{6}R + D\right)\,. \tag{3.7}$$

Upon adding this part to the lagrangian (3.2) the D-field equation of motion yields the relation

$$-i\left(F_\Lambda \bar{X}^\Lambda - \bar{F}_\Lambda X^\Lambda\right) = 2\mathcal{F}_{IJ}\left(v^I\bar{v}^J + \frac{1}{4}x^I x^J\right)\,. \tag{3.8}$$

Substituting this back into (3.5) results in an Einstein-Hilbert term of the form

$$e^{-1}\,\mathcal{L} = -R\,\mathcal{F}_{IJ}\left(v^I\bar{v}^J + \frac{1}{4}x^I x^J\right)\,. \tag{3.9}$$

[6]Since we are interested in quantum corrections to the hypermultiplet sector, we will ignore the vector multiplet geometry in the following and include the terms required for gauge fixing the superconformal tensor multiplet theory only. It is, however, straightforward to also include the complete vector multiplet sector in the setup.

In order to have a canonical normalization we choose the dilatation gauge

$$2\,\kappa^2\,\mathcal{F}_{x^I x^J}\left(v^I\bar{v}^J+\frac{1}{4}x^I x^J\right)=1\,. \tag{3.10}$$

Eliminating the fields $\vec{\mathcal{V}}_\mu$ and D together with imposing the gauge constraints (3.6) and (3.10) defines the superconformal quotient of the lagrangian (3.2). The resulting theory is a Poincaré supergravity theory coupled to n_H-1 tensor multiplets and one double tensor multiplet whose scalars parameterize the manifold $\mathcal{M}_{\rm TM}$. The corresponding lagrangian reads

$$\begin{aligned}e^{-1}\mathcal{L}=&-\frac{1}{2\kappa^2}R+\mathcal{F}_{x^I x^J}\left(\partial_\mu v^I\partial^\mu\bar{v}^J+\frac{1}{4}\partial_\mu x^I\partial^\mu x^J-E^I_\mu\,E^{J\mu}\right)\\&-\frac{1}{2}\vec{S}_\mu M^{-1}\vec{S}^\mu-\frac{1}{2}\vec{T}_\mu M^{-1}\vec{T}^\mu-\vec{S}_\mu M^{-1}\vec{T}^\mu\\&+i\,E^I_\mu\left(\mathcal{F}_{v^I x^J}\,\partial^\mu v^J-\mathcal{F}_{\bar{v}^I x^J}\partial^\mu\bar{v}^J\right)\,,\end{aligned} \tag{3.11}$$

where the constraints (3.6) and (3.10) are implicitly understood and M^{-1} is the inverse of the matrix (3.5). Henceforth we will work in the conventions where $\kappa^{-2}=2$.

Taking the lagrangian (3.11) and dualizing the tensor fields into scalars by adding a suitable Lagrange multiplier finally leads to a standard Poincaré supergravity theory coupled to n_H hypermultiplets. The scalars parameterize the manifold $\mathcal{M}_{\rm HM}$ which for $n_H=n_V+1$ has precisely the correct dimension for a manifold in the image of the c-map. Note that the lagrangian (3.11) is also completely determined by $\mathcal{F}(v,\bar{v},x)$ which in turn is fixed by the function $H(\eta)$ appearing in the contour integral (2.9).

3.2 The classical c-map

Before embarking on the computation of the perturbatively corrected hypermultiplet moduli space, we need to connect the classical result (2.12) to the hypermultiplet lagrangian (2.4), using the formalism detailed in the previous subsection. This computation provides the dictionary between the variables $x^0, x^\Lambda, v^\Lambda$ appearing on the superconformal tensor side and the HM scalars coming from the classical string compactification. In particular this will identify x^0 as the dilaton which then controls the perturbation series set up in the next section. Our computation thereby completely parallels the one for the one-loop corrections presented in appendix A from which all intermediate results may be obtained by setting $c=D(z)=\bar{D}(\bar{z})=0$.

3.2.1 Gauge-fixing the superconformal symmetries

Our starting point is the function $H(\eta)$ encoding the classical superconformal c-map (2.12) which is then substituted into the contour integral (2.9). To evaluate this contour explicitly, we perform a partial gauge-fixing of the $\mathrm{SU}(2)/\,\mathrm{U}(1)\subset\mathrm{SU}(2)$ symmetries that belong to the superconformal symmetry group. A convenient gauge-choice is taken by setting $v^0=0$.[7]

[7] In the context of the superconformal quotient for hyper-Kähler cones this corresponds to gauge fixing a coordinate on the twistor space.

The partially gauge fixed function $\mathcal{F}$ determining the superconformal TM lagrangian is then given by [26]

$$\mathcal{F}(x^0, v^\Lambda, \bar{v}^\Lambda, x^\Lambda) = -\text{Im}\left[\frac{1}{x^0}\oint_{\mathcal{C}} \frac{\mathrm{d}\zeta}{2\pi i\,\zeta}\, F(\eta^\Lambda)\right], \tag{3.12}$$

where F is the holomorphic prepotential encoding the VM couplings of the dual type II compactification and $\mathcal{C}$ is a positively oriented contour around the origin. This integral can be evaluated explicitly by using the homogeneity property $F(\eta) = \frac{1}{\zeta^2} F(\zeta\eta)$ and assuming that $F(\zeta\eta)$ has no poles at the origin of the complex ζ-plane. This yields

$$\mathcal{F}(x^0, v^\Lambda, \bar{v}^\Lambda, x^\Lambda) = \frac{1}{4x^0}\left(N_{\Lambda\Sigma} x^\Lambda x^\Sigma - 2K(v, \bar{v})\right), \tag{3.13}$$

where $K(v, \bar{v})$ and $N_{\Lambda\Sigma}$ are the objects from special geometry defined in (2.8), but now evaluated in terms of the tensor multiplet scalars v^Λ.

In the next step we compute the derivatives of $\mathcal{F}(x^0, v^\Lambda, \bar{v}^\Lambda, x^\Lambda)$ entering into the lagrangian (3.2). For the derivatives which do not involve the coordinates $v^0, \bar{v}^0$ this is straightforward. Writing the result in terms of the inhomogeneous coordinates

$$A^\Lambda \equiv \frac{x^\Lambda}{2\,x^0}, \qquad z^\Lambda \equiv \frac{v^\Lambda}{v^1}, \tag{3.14}$$

the resulting expressions are readily be obtained from eqs. (A.2) by setting $c = 0$. One can then verify that these equations satisfy all those constrains in (2.11) which do not involve derivatives with respect to v^0. The remaining conditions cannot be checked since the partially gauge fixed result (3.13) does not allow to compute derivatives of $\mathcal{F}(x^0, v^\Lambda, \bar{v}^\Lambda, x^\Lambda)$ with respect to v^0. This is, however, not an obstacle when constructing the TM lagrangian as the derivatives $\mathcal{F}_{x^0 v^0}, \mathcal{F}_{x^\Lambda v^0}$ drop out from (3.11) due to setting $v^0 = 0 \to \partial_\mu v^0 = 0$.

We now fix the remaining superconformal gauge symmetries. In order to break the residual U(1) and dilatation symmetries which are left after imposing $v^0 = 0$, we set $v^1 = \bar{v}^1$, implementing the gauge choice (3.6). Furthermore, we have to solve the embedding equation (3.10) that arises after fixing the dilatations. Substituting the derivatives of $\mathcal{F}$ and using the homogeneity property of the Kähler potential (2.8) the condition (3.10) can easily be solved for v^1:

$$v^1 = \sqrt{\frac{4x^0}{K(z, \bar{z})}}. \tag{3.15}$$

Here $K(z, \bar{z}) = z^\Lambda N_{\Lambda\Sigma} \bar{z}^\Sigma$ is understood as a function of the inhomogeneous coordinates $z, \bar{z}$, eq. (3.14). This relation then expresses $v^1 = \bar{v}^1$ in terms of the coordinates $x^0, z^a, \bar{z}^a$. Together with the gauge condition (3.6), eq. (3.15) completely fixes the superconformal symmetries on the tensor side.

3.2.2 The tensor multiplet lagrangian

Following the general construction outlined in the previous subsection we now calculate the inverse of the matrix M, eq. (3.5), and the SU(2)-currents $\vec{S}_\mu$ and $\vec{T}_\mu$ given in (3.3) and (3.4) taking the gauge choices (3.6) and (3.15) into account. Again the resulting expressions are easily obtained from eqs. (A.8), (A.5) and (A.6) by setting $\Delta = 0$. Substituting these results into the lagrangian (3.11) gives the TM lagrangian for the classical case

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm cl}^{\rm TM} = & -\frac{1}{2(x^0)^2}(\partial_\mu x^0)^2 + 2\,\mathcal{G}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \left(\frac{x^0}{2}\right)(\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma}\partial_\mu A^\Lambda \partial^\mu A^\Sigma \\ & + 2\,\mathcal{T}_{IJ}^{\rm cl}\, E_\mu^I E^{J\mu} + i(\mathcal{N}-\bar{\mathcal{N}})_{\Lambda\Sigma}\left[(\partial_\mu A^\Lambda)E^{\Sigma\mu} - 2(\partial_\mu A^\Lambda)A^\Sigma E^{0\mu}\right] . \end{aligned} \tag{3.16}$$

Here $\mathcal{G}_{a\bar{b}}$ and $\mathcal{N}_{\Lambda\Sigma}$ are given in (2.2) and (2.3), respectively, and the matrix $\mathcal{T}_{IJ}^{\rm cl}$ appearing in the tensor field kinetic term is given by

$$\mathcal{T}_{IJ}^{\rm cl} = \frac{1}{x^0}\begin{bmatrix} \frac{1}{x^0} - (\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma}A^\Lambda A^\Sigma & \frac{1}{2}(\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma}A^\Lambda \\ \frac{1}{2}(\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma}A^\Sigma & -\frac{1}{4}(\mathcal{N}+\bar{\mathcal{N}})_{\Lambda\Sigma} \end{bmatrix} . \tag{3.17}$$

Upon setting $x^0 = {\rm e}^{-\phi}$ this is precisely the classical tensor multiplet lagrangian (2.6). This identification only makes sense for units in which $\kappa^{-2} = 2$. One can reinstall $(2\kappa^2)^{-1}$ as an overall factor in front of the lagrangian, and in this convention all the fields are dimensionless. In particular, our four-dimensional dilaton is dimensionless and is related to the string coupling constant as

$${\rm e}^{-\phi_\infty/2} = g_s \ . \tag{3.18}$$

This relation is up to (dimensionless) volume factors of the CY_3, but we will work in conventions in which we set this to unity. They are not important for counting powers of g_s. The result of our gauge-fixing condition (3.15) implies that v^1 also scales like g_s. This is consistent with the observation made at the end of section 2, where we say that η^1/η^0 scales like g_s^{-1}.

3.2.3 The dual hypermultiplet lagrangian

Finally, we construct the HM lagrangian dual to (3.16) by converting the tensor into scalar fields. For this purpose we add the Lagrange multipliers

$$e^{-1}\mathcal{L}^{\rm LM} = 2\,(\partial_\mu w_0)\,E^{0\mu} - (\partial_\mu B_\Lambda)\,E^{\Lambda\mu} \tag{3.19}$$

to the TM lagrangian (3.16). Here the prefactors are purely conventional and have been chosen for later convenience. We then eliminate the tensor field strength in favor of the scalars w_0, B_Λ by substituting their algebraic equation of motion back into $\mathcal{L}_{\rm cl}^{\rm TM} + \mathcal{L}^{\rm LM}$. This results in the classical HM lagrangian

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm cl}^{\rm HM} = & -\frac{1}{2(x^0)^2}(\partial_\mu x^0)^2 + 2\,\mathcal{G}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} \\ & + \frac{1}{2}x^0\,(\mathcal{N}+\bar{\mathcal{N}})^{\Lambda\Sigma}\left|2\mathcal{N}_{\Lambda\Xi}\partial_\mu A^\Xi + i\partial_\mu B_\Lambda\right|^2 - \frac{1}{2}(x^0)^2\left(\partial_\mu w_0 - A^\Lambda\partial_\mu B_\Lambda\right)^2 . \end{aligned} \tag{3.20}$$

Comparing this expression to the one obtained by the classical Poincaré c-map (2.4) we find complete agreement after identifying

$$x^0 = {\rm e}^{-\phi}\,, \qquad w_0 = \sigma + \frac{1}{2}A^\Sigma B_\Sigma\,. \tag{3.21}$$

Furthermore, eq. (3.21) allows us to determine the action of the Peccei-Quinn isometries (2.5) on the coordinates $x^0, w_0, A^\Lambda, B_\Lambda, z^\Lambda$ and $\bar{z}^\Lambda$:

$$\delta w_0 = \delta\epsilon + \alpha^\Sigma B_\Sigma \,, \qquad \delta A^\Lambda = \alpha^\Lambda \,, \qquad \delta B_\Lambda = \beta_\Lambda \,. \tag{3.22}$$

By using the definition (2.10), and comparing (3.14) to (3.21) the action of the Peccei-Quinn isometries can then be implemented directly in superspace. While the shifts associated with ϵ and β_Λ are automatically encoded in the TM description, α^Λ acts non-trivially on the scalars η^I [27]:

$$\eta^0 \to \eta^0 \,, \qquad \eta^\Lambda \to \eta^\Lambda + 2\alpha^\Lambda \eta^0 \,. \tag{3.23}$$

It is instructive, and will be useful in the next section, to check directly in superspace that (3.12) leads to a lagrangian which is invariant under (3.23). While (3.23) is a gauge-independent statement, we verify this invariance in the particular gauge $v^0 = 0$. The infinitesimal variation of (3.12) under (3.23) then gives

$$\delta\left[-\mathrm{Im}\left(\frac{1}{x^0}\oint_{\mathcal{C}}\frac{\mathrm{d}\zeta}{2\pi i\,\zeta}\,F(\eta)\right)\right] = -\mathrm{Im}\left[2\alpha^\Lambda\, x^\Sigma\, F_{\Lambda\Sigma}(v)\right] \,. \tag{3.24}$$

It is straightforward to check that substituting the second derivatives of this variation into the lagrangian (3.11) results in a total derivative in the action. The latter is then invariant under the Peccei-Quinn symmetries (3.23). Equipped with this knowledge we are now ready to discuss the string loop corrections to the classical lagrangian (3.20).

4. One-loop corrections

In order to identify the perturbatively corrected HM moduli space, we follow the strategy of [19] and search for deformations of the classical result (3.20) compatible with string perturbation theory, i.e., satisfying the conditions (1) - (6) given in the introduction. The superconformal quotient construction of section 3 shifts the issue of implementing the QK condition (1) to finding suitable deformations of $H^{\mathrm{cl}}(\eta)$ which obey the conditions (2) - (6). Our construction proceeds as follows: first we determine deformations of $H^{\mathrm{cl}}(\eta)$ which are subleading in the dilaton and preserve the Peccei-Quinn symmetries in Subsection 4.1 thereby implementing the conditions (1) - (4). We then construct the corresponding HM lagrangian via the superconformal quotient construction before incorporating the remaining conditions (5) - (6). This procedure will lead to the perturbatively corrected HM spaces given at the end of Subsection 4.4.

4.1 Deformations of $H^{\mathrm{cl}}(\eta)$

We start by investigating deformations of $H^{\mathrm{cl}}(\eta) = \frac{F(\eta^\Lambda)}{\eta^0}$. The QK condition (1) enforces that such deformations are homogeneous of degree one under a rescaling of η^I and have no explicit ζ dependence (cfg. Subsection 2.2). Next we demand that the deformations should be subleading in g_s, i.e. they are at least of order g_s^2 higher than the classical result. The powers of g_s can be counted using the fact that η^1/η^0 scales like g_s^{-1}. Moreover, in the gauge $v^0 = 0$, $\eta^0 = x^0 = \mathrm{e}^{-\phi}$ is of order g_s^2. From this one can see that for the universal hypermultiplet, the first term in (2.13) is of order g_s^0 while the second is of order g_s^2, which is the correct counting for the one-loop correction. These considerations naturally suggests the following candidates for correction terms to $H^{\rm cl}(\eta)$:[8]

$$\begin{aligned} H_1^{\rm def}(\eta) &= D_1(\eta^\Lambda)\,, && (4.1)\\ H_2^{\rm def}(\eta) &= D_2(\eta^\Lambda)\,\ln(\eta^0)\,, && (4.2)\\ H_3^{\rm def}(\eta) &= D_3(\eta^\Lambda)\,\eta^0\,, && (4.3)\\ H_4^{\rm def}(\eta) &= D_4(\eta^\Lambda)\,\eta^0\,\ln(\eta^0)\,. && (4.4)\end{aligned}$$

Here the functions $D_1(\eta^\Lambda)$ and $D_2(\eta^\Lambda)$ are homogeneous of degree one while $D_3(\eta^\Lambda)$ and $D_4(\eta^\Lambda)$ are homogeneous of degree zero.

Let us now discuss how these deformations contribute to the superconformal TM lagrangian (3.11). In this course we compute their contribution to the function $\mathcal{F}(x, v, \bar{v})$ by evaluating the contour integral (2.9) for the same gauge and contour as in the classical case. Substituting the second derivatives of the resulting functions $\mathcal{F}(x, v, \bar{v})$ into the TM lagrangian then shows that the deformations (4.1) and (4.3) give rise to surface terms only. Furthermore, a careful analysis of the contributions arising from the deformation (4.2) yields that the corresponding terms contain an odd number of RR fields. Thus (4.2) violates condition (4) and *is not compatible* with string perturbation theory.

Hence we are left with considering the deformation (4.4). As it will turn out in the subsequent subsections this deformation indeed satisfies the remaining conditions (3)-(6). Denoting $D_4(v) = -4cD(v)$, where the numerical factor $-4c$ has been extracted for later convenience, we thus make the following ansatz for the "loop-corrected" function $H^{\rm qc}(\eta^I)$:

$$H^{\rm qc}(\eta^I) = \frac{F(\eta^\Lambda)}{\eta^0} - 4\,c\,D(\eta^\Lambda)\,\eta^0\,\ln\,\eta^0\,. \tag{4.5}$$

We then proceed to the evaluation of the contour integral (2.9) with $H^{\rm qc}(\eta)$. As in the classical case, the integral is carried out in the gauge $v^0 = \bar{v}^0 = 0$ and with the contour $\mathcal{C}$ taken around the origin. Assuming that $D(\zeta\eta^\Lambda)$ has no poles at $\zeta = 0$ the computation is straightforward and yields

$$\mathcal{F}(x^0, v^\Lambda, \bar{v}^\Lambda, x^\Lambda) = \frac{1}{4x^0}\left(N_{\Lambda\Sigma}x^\Lambda x^\Sigma - 2K(v, \bar{v})\right) - 2ic\left[D(v^\Lambda) - \bar{D}(\bar{v}^\Lambda)\right]\,x^0\,\ln\,(x^0)\,. \tag{4.6}$$

At this stage we are just left with verifying condition (3), namely that (4.5) leads to an action which is invariant under the Peccei-Quinn transformations (3.23). Since the classical piece already respects these symmetries (cfg. (3.24)), we need to check the deformation piece only. Under the variation (3.23), eq. (4.4) becomes

$$\delta\mathcal{F}_4^{\rm def}(v, \bar{v}, x) = 8\,c\,\alpha^\Lambda{\rm Im}\oint_{\mathcal{C}}\frac{{\rm d}\zeta}{2\pi i}\left[D_\Lambda(\zeta\eta^\Sigma)(\eta^0)^2\,\ln\,(\eta^0)\right]\,. \tag{4.7}$$

[8]Here and in the following we will not consider terms containing higher powers of $\ln(\eta^0)$. Such terms typically violate the homogeneity properties. Moreover, they give rise to terms polynomial in $\phi = -\ln({\rm e}^{-\phi})$ in the effective action, which do not occur in string perturbation theory. Deformations containing higher powers of η^0 will be discussed in section 5. Deformations with lower powers (i.e. negative powers) in η^0 are not consistent with the Peccei-Quinn isometries (3.23).

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This vanishes as long as $D_\Lambda(\zeta\eta)$ has no poles around the origin. With this proviso, (4.5) will lead to quaternion-Kähler metrics invariant under the perturbative Peccei-Quinn symmetries.

Let us end this subsection with a remark about the universal hypermultiplet. It was found in [20] that the one-loop corrected HM lagrangian [19] can be encoded in $H^{\mathrm{UHM}}(\eta)$, eq. (2.13). We observe that the ansatz (4.5) naturally reduces to this equation if we set $D(v) = -i$. Up to a rescaling by a constant (which can be absorbed in c) this is, however, the only non-trivial $D(v)$ allowed in the absence of non-universal hypermultiplets. Thus our ansatz naturally generalizes the result for the universal hypermultiplet.

4.2 The deformed tensor multiplet lagrangian

We now proceed by computing the perturbative corrections to the classical TM lagrangian which arise from (4.6). Since this calculation is lengthy and rather technical its details are given in appendix A. Here we just quote the final result for the deformed TM lagrangian. With the same conventions as in (3.14) and (3.21), we have

$$\begin{aligned} e^{-1}\mathcal{L}^{\mathrm{TM}}_{\mathrm{qc}} = &- \frac{1+2\Delta \mathrm{e}^{-\phi}}{2(1+\Delta\mathrm{e}^{-\phi})}(\partial_\mu\phi)^2 + 2(1+\Delta\mathrm{e}^{-\phi})\mathcal{G}_{a\bar{b}}\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}} + \frac{c^2\,\mathrm{e}^{-2\phi}}{8(1+\Delta\mathrm{e}^{-\phi})}\tilde{A}_\mu\tilde{A}^\mu \\ &+ \frac{\mathrm{e}^{-\phi}}{2}(\mathcal{M}+\bar{\mathcal{M}})_{\Lambda\Sigma}\partial_\mu A^\Lambda\partial^\mu A^\Sigma + 2\mathcal{T}^{\mathrm{qc}}_{IJ}E^I_\mu E^{J\mu} \\ &+ i\left(\mathcal{M}-\bar{\mathcal{M}}\right)_{\Lambda\Sigma}\left[E^\Lambda_\mu\partial^\mu A^\Sigma - 2E^0_\mu A^\Lambda\partial^\mu A^\Sigma\right] \\ &- 2\Delta A_\mu E^{0\mu} + 2c\left[D(z)+\bar{D}(\bar{z})\right](\partial_\mu\phi)E^{0\mu}\ . \end{aligned} \tag{4.8}$$

In this expression, we have introduced the notation

$$\Delta = \frac{ic}{2}\left[D(z) - \bar{D}(\bar{z})\right], \tag{4.9}$$

$\mathcal{G}_{a\bar{b}}(z,\bar{z})$ is the local special Kähler metric (2.2) arising from the prepotential of the dual VM geometry, and the matrix $\mathcal{M}_{\Lambda\Sigma}$ appearing in the kinetic term of the RR scalars is given by

$$\mathcal{M}_{\Lambda\Sigma}(z,\bar{z},\phi) = -i\bar{F}_{\Lambda\Sigma} - \frac{(Nz)_\Lambda(Nz)_\Sigma}{(zNz)}\Big(1+\mathrm{e}^{-2\phi}\frac{\Delta^2}{|\tilde{M}|}\Big) - \frac{\Delta\,|v^1|^2}{4|\tilde{M}|}(Nz)_\Lambda(N\bar{z})_\Sigma\ . \tag{4.10}$$

The factor of $|v^1|^2$ appears as a result of our gauge fixing procedure. Similar to (3.15), we set v^1 to be real and solve (3.10) at the one-loop level,

$$v^1 = \sqrt{\frac{4x^0(1+\Delta x^0)}{K(z,\bar{z})}}\,. \tag{4.11}$$

The quantity $|\tilde{M}|$ is related to the determinant of the matrix M appearing in the tensor multiplet lagrangian (3.11). Applied to our situation, it is given by

$$|\tilde{M}| = \frac{\mathrm{e}^{2\phi}|v^1|^4}{16}|z^\Lambda N_{\Lambda\Sigma}z^\Sigma|^2 - \mathrm{e}^{-2\phi}\Delta^2\ . \tag{4.12}$$

It is clear that in the classical limit, $\Delta = 0$, the matrix $\mathcal{M}$ coincides with $\mathcal{N}$ defined in (2.3). Furthermore, the kinetic term for the tensor fields is determined through

$$\mathcal{T}_{IJ}^{\rm qc} = \begin{bmatrix} f e^{2\phi} + 4\mathcal{T}_{\Lambda\Sigma}^{\rm qc} A^\Lambda A^\Sigma & -2\mathcal{T}_{\Sigma\Xi}^{\rm qc} A^\Xi \\ -2\mathcal{T}_{\Lambda\Xi}^{\rm qc} A^\Xi & \mathcal{T}_{\Lambda\Sigma}^{\rm qc} \end{bmatrix} , \qquad f = \frac{1 + 2\Delta\, e^{-\phi}}{1 + \Delta\, e^{-\phi}} , \tag{4.13}$$

where

$$-4e^{-\phi}\mathcal{T}_{\Lambda\Sigma}^{\rm qc} = -i\bar{F}_{\Lambda\Sigma} - \frac{(Nz)_\Lambda (Nz)_\Sigma}{(zNz)}\Big(1 + e^{-2\phi}\frac{\Delta^2}{|\tilde{M}|}\Big) + \frac{\Delta\,|v^1|^2}{4|\tilde{M}|}(Nz)_\Lambda (N\bar{z})_\Sigma + \text{h.c.} \,. \tag{4.14}$$

Notice the similarity with (2.7) to which it reduces in the classical limit. Finally, A_μ and $\tilde{A}_\mu$ are given by the Kähler connection

$$A_\mu = \frac{i}{K(z,\bar{z})}\left[(N\bar{z})_a \partial_\mu z^a - (Nz)_{\bar{a}} \partial_\mu \bar{z}^{\bar{a}}\right] , \tag{4.15}$$

and

$$\tilde{A}_\mu = i\left[\partial_a D(z)\partial_\mu z^a - \partial_{\bar{a}}\bar{D}(\bar{z})\partial_\mu \bar{z}^{\bar{a}}\right] , \tag{4.16}$$

respectively.

Let us now comment about the structure of the one-loop corrected lagrangian (4.8) and contrast it with the classical expression (3.16). We first of all remark that the dilaton kinetic term is modified. Comparing to the universal hypermultiplet (A.43) this term has the same functional dependence on the dilaton, but in the generic case the constant c can be promoted to a function of the geometric moduli $z, \bar{z}$. The terms involving the squares of the RR scalars and the tensors have the same structure as in the classical case, but the classical couplings $\mathcal{N}$ and $\mathcal{T}^{\rm cl}$ are now replaced by their "quantum corrected" counterparts $\mathcal{M}$ and $\mathcal{T}^{\rm qc}$. However, due to the different sign in the last term, it is not the combination $\mathcal{M} + \bar{\mathcal{M}}$ which enters into $\mathcal{T}^{\rm qc}$, as one might have expected from (3.17). In fact this sign difference is necessary for obtaining the quantum corrected universal hypermultiplet in appendix A.4.

The metric for the geometric moduli $z, \bar{z}$ receives two kinds of loop-corrections: the first is the fiberwise rescaling by $(1 + \Delta e^{-\phi})$ along the dilaton direction (possibly with z-dependent corrections to the metric, encoded in Δ). The second kind of corrections (third term in the first line of (4.8)) depends quadratically on $\tilde{A}_\mu$ and induces explicit non-Kähler $\partial z \partial z$, $\partial\bar{z}\partial\bar{z}$ terms, in addition to further correcting the mixed terms. These terms disappear however when $D(z)$ is constant. Finally, there are also the quantum mixing terms in the last line of (4.8).

Let us end this subsection by noting that upon setting $c = \Delta = 0$ the quantum corrected TM lagrangian (4.8) reduces to the classical result (3.16), consistently with turning off the deformation in (4.6). See appendix A.2 for details.

4.3 The deformed hypermultiplet lagrangian

With the result (4.8) at hand we now compute the corresponding HM lagrangian by dualizing the tensors E_μ^0, E_μ^Λ to scalars w_0, B_Λ. Again, the details of the dualization can be found in appendix A. The final result for the deformed HM lagrangian then reads

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm HM}_{\rm qc} =& -\frac{1+2\Delta\,\mathrm{e}^{-\phi}}{2(1+\Delta\,\mathrm{e}^{-\phi})}(\partial_\mu\phi)^2 + 2(1+\Delta\mathrm{e}^{-\phi})\mathcal{G}_{a\bar{b}}\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}} + \frac{c^2\,\mathrm{e}^{-2\phi}}{8(1+\Delta\mathrm{e}^{-\phi})}\tilde{A}_\mu\tilde{A}^\mu \\ & - \frac{\mathrm{e}^{-2\phi}(1+\Delta\,\mathrm{e}^{-\phi})}{2(1+2\Delta\,\mathrm{e}^{-\phi})}\left|\partial_\mu w_0 - \Delta A_\mu + c\left[D(z)+\bar{D}(\bar{z})\right](\partial_\mu\phi) - A^\Lambda\partial_\mu B_\Lambda\right|^2 \\ & - \frac{1}{8}(\mathcal{T}^{\rm qc})^{\Lambda\Sigma}\left(2\mathcal{M}_{\Lambda\Xi}\partial^\mu A^\Xi + i\partial^\mu B_\Lambda\right)\left(2\bar{\mathcal{M}}_{\Sigma\Xi}\partial_\mu A^\Xi - i\partial_\mu B_\Sigma\right) \\ & - \mathrm{Re}\left[\Delta\mathrm{e}^{-\phi}\frac{(1+\Delta\mathrm{e}^{-\phi})}{2|\tilde{M}|K}(Nz)_\Lambda(N\bar{z})_\Xi(\mathcal{T}^{\rm qc})^{\Xi\Pi}(\mathcal{M}+\bar{\mathcal{M}})_{\Pi\Sigma}\partial_\mu A^\Lambda\partial^\mu A^\Sigma\right]\,. \end{aligned} \tag{4.17}$$

Here $(\mathcal{T}^{\rm qc})^{\Lambda\Sigma}$ is the inverse of (4.14). An alternative way of writing this lagrangian is given in (A.31).

One should emphasize that this family of metrics is QK *by construction.*[9] Furthermore, it is completely specified by *two* holomorphic functions, the prepotential $F(v)$ (homogeneous of degree two) determining the classical result and $cD(v)$ (homogeneous of degree zero) encoding the "quantum deformations". In order to establish that these quantum deformations indeed describe string loop corrections to the classical HM moduli space, we still have to require that the metrics (4.17) satisfy the conditions (5)-(6) from the introduction. We will then implement the requirement (5) here, and leave (6) for the next subsection.

Concerning condition (5), we use that parity acts on the fields appearing in (4.17) by

$$w_0 \leftrightarrow -w_0\,, \qquad B_\Lambda \leftrightarrow -B_\Lambda\,, \qquad A^\Lambda \leftrightarrow A^\Lambda\,, \qquad z^\Lambda \leftrightarrow \bar{z}^\Lambda\,. \tag{4.18}$$

Applying that $F_{\Lambda\Sigma}(z) = -\bar{F}_{\Lambda\Sigma}(z)$, this transformation is a symmetry of the lagrangian if

$$D(z) = -\bar{D}(z)\;. \tag{4.19}$$

Thus invariance under parity imposes the restriction that $D(z)$ is purely imaginary. We further remark that the RR scalars A^Λ and B_Λ enter into (4.17) in pairs only so that (4) is satisfied automatically.

4.4 Matching to string loop amplitudes

Finally, let us compare the lagrangian (4.17) to known results on string loop-corrected HM metrics, implementing condition (6). The only undetermined quantities of our lagrangian are a numerical constant c, and a holomorphic function $D(z)$. We will fix them by comparing to known results for string loop amplitudes.

We start with the sector of the universal hypermultiplet. This can be obtained from compactifications of IIA strings on a rigid ($h^{1,2}=0$) CY_3, so there are no geometric moduli z^a. The one-loop correction was determined in [19] and it suffices to look at the kinetic term of the dilaton only. In Einstein frame, the one-loop correction can be written as

$$e^{-1}\mathcal{L}^{\rm UHM} = -\frac{1}{2}\,\frac{(1-2\chi_1\mathrm{e}^{-\phi})}{(1-\chi_1\mathrm{e}^{-\phi})}\,(\partial_\mu\phi)^2 + \cdots\;. \tag{4.20}$$

[9] As a non-trivial consistency check, we used Mathematica to explicitly verify the Einstein property of these metrics in several examples including two hypermultiplets.

Here, ϕ is related to the four-dimensional dilaton in a way explained in [19], and the one-loop constant χ_1 is proportional to the Euler number $\chi = 2(h^{1,1} - h^{1,2})$ of the CY_3. The numerical value of χ_1 was determined in [19], generalizing previous results in [45],

$$\chi_1 = \frac{4\zeta(2)\chi}{(2\pi)^3} = \frac{1}{6\pi}(h^{1,1} - h^{1,2}) \; . \tag{4.21}$$

Comparing now to our general effective action (4.17), we determine that

$$c = -\chi_1 = -\frac{\chi}{12\pi} \; . \tag{4.22}$$

Here we have used that the function D in (4.9) must be an imaginary constant, since there are no geometric moduli for rigid Calabi-Yau's. We have normalized it such that $D = -i$.

We now look at the generic compactifications, and consider the cases of IIA and IIB separately. Because we have a common description for universal and non-universal hypermultiplets, the value of c is fixed and also appears in the one-loop corrections to the kinetic terms of the geometric moduli z^a. This sector has been considered in [18], and in Einstein frame these corrections were found to take the form

$$S_{\mathrm{IIA}} = \frac{1}{2\kappa_4^2} \int \mathrm{d}^4x \sqrt{-g^{(E)}}\, 2\left(1 + \chi_2 \mathrm{e}^{-\phi}\right) \mathcal{G}^0_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} \, , \tag{4.23}$$

for type IIA, and

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_4^2} \int \mathrm{d}^4x \sqrt{-g^{(E)}}\, 2\left(\mathcal{G}_{a\bar{b}} - \chi_2 \mathrm{e}^{-\phi}\left(\mathcal{G}^0_{a\bar{b}} + \cdots\right)\right) \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} \, , \tag{4.24}$$

for type IIB. Here, χ_2 is a parameter that characterizes the one-loop correction. Furthermore, the special Kähler metrics $\mathcal{G}_{a\bar{b}}$ and $\mathcal{G}^0_{a\bar{b}}$ are computed from the prepotential[10]

$$F(X) = \frac{1}{3!}\kappa_{abc}\frac{X^a X^b X^c}{X^1} - i\frac{\zeta(3)\chi}{2(2\pi)^3}(X^1)^2 \, , \tag{4.25}$$

where $\mathcal{G}_{a\bar{b}}$ arises from the full prepotential (including perturbative α' corrections encoded in the second term) while $\mathcal{G}^0_{a\bar{b}}$ comes from the first term only. This implies that, at tree level in g_s, the α' corrections occurring in the type IIB case are encoded in $\mathcal{G}_{a\bar{b}}$ while possible α' corrections at one-loop are contained in the dots. Notice that these corrections are absent in the type IIA case. Additionally it was shown that eqs. (4.23) and (4.24) can be understood from the dimensional reduction of the ten-dimensional effective actions on a CY_3 if the R^4 terms are taken into account. Finally, we note that the one-loop corrections in type IIA and type IIB come with a different sign. This is consistent with mirror symmetry (now at one-loop in g_s) which sends $\chi \leftrightarrow -\chi$ and exchanges IIA $\leftrightarrow$ IIB.

After these preliminaries, we are now ready to determine the one-loop deformation $cD(z)$ by comparing the lagrangian (4.17) to (4.23) and (4.24) in the type IIA and IIB case, respectively. Considering the type IIA case we observe that, consistently with four-dimensional $\mathcal{N} = 2$ supersymmetry, the corrections to the HM metric arise entirely in g_s.

[10] On top of the perturbative corrections, $F(X)$ receives contributions from worldsheet instantons, but we refrain from giving explicit expressions.

Comparing the appropriate subsectors of (4.17) and (4.23) leads to $\Delta = \chi_2$. Recalling eq. (4.9) this amounts to having $D(z)$ equal to (a purely imaginary) constant, which we normalize to be $-i$. We then have

$$D_{\rm IIA} = -i\,, \qquad c = \chi_2. \tag{4.26}$$

But from the universal sector, we had already concluded that $c = -\chi_1$. Therefore, χ_2 is fixed by supersymmetry to be $\chi_2 = -\chi_1$. The value of χ_2 found in [18] differs from ours by a factor of -2. It would be interesting to resolve this apparent mismatch.

The IIB case is more complicated due to the presence of α'-corrections. First observe that $\Delta = (ic/2)\left[D(z) - \bar{D}(\bar{z})\right]$ is the sum of a holomorphic and anti-holomorphic function. Clearly, it is implausible that volume dependent terms, with

$$\mathcal{V} = -\frac{i}{6}\kappa_{abc}(z^a - \bar{z}^a)(z^b - \bar{z}^b)(z^c - \bar{z}^c)\,, \tag{4.27}$$

separate into the sum of holomorphic and anti-holomorphic expressions in such a way that they can contribute to Δ. Comparing to the expression (4.17) this implies that the dots appearing in (4.23) complete $\mathcal{G}^0_{a\bar{b}}$ into the full α'-corrected metric $\mathcal{G}_{a\bar{b}}$. As a consequence (4.23) becomes

$$S_{\rm IIB} = \frac{1}{2\kappa_4^2}\int {\rm d}^4x\sqrt{-g^{(E)}}\left(1 - {\rm e}^{-\phi}\chi_2\right)\mathcal{G}_{a\bar{b}}\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}}\ . \tag{4.28}$$

From this we then read off

$$D_{\rm IIB} = i\,, \qquad c = \chi_2\ . \tag{4.29}$$

A constant $D(z)$ leads to further simplifications of the expression (4.17). Thus we can state our final result for the type IIA compactification as (for IIB, change c into $-c$):

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm HM}_{\rm qc} =& -\frac{1 + 2c\,{\rm e}^{-\phi}}{2(1 + c\,{\rm e}^{-\phi})}(\partial_\mu\phi)^2 + 2(1 + c\,{\rm e}^{-\phi})\mathcal{G}_{a\bar{b}}\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}} \\ &- \frac{{\rm e}^{-2\phi}(1 + c\,{\rm e}^{-\phi})}{2(1 + 2c\,{\rm e}^{-\phi})}\left|\partial_\mu w_0 - cA_\mu - A^\Lambda\partial_\mu B_\Lambda\right|^2 \\ &- \frac{1}{8}(\mathcal{T}^{\rm qc})^{\Lambda\Sigma}\left(2\mathcal{M}_{\Lambda\Xi}\partial^\mu A^\Xi + i\partial^\mu B_\Lambda\right)\left(2\bar{\mathcal{M}}_{\Sigma\Xi}\partial_\mu A^\Xi - i\partial_\mu B_\Sigma\right) \\ &- {\rm Re}\left[c\,{\rm e}^{-\phi}\frac{(1 + c\,{\rm e}^{-\phi})}{2|\tilde{M}|K}(Nz)_\Lambda(N\bar{z})_\Xi(\mathcal{T}^{\rm qc})^{\Xi\Upsilon}(\mathcal{M} + \bar{\mathcal{M}})_{\Upsilon\Sigma}\partial_\mu A^\Lambda\partial^\mu A^\Sigma\right]\ . \end{aligned} \tag{4.30}$$

We remark that, for the type IIA case, the appearance of the Kähler connection in the second line of (4.30) was already anticipated in [21], based on an analogy to heterotic strings.

5. Higher loop corrections

A natural question is to ask about the presence of higher loop corrections. For the case of the universal hypermultiplet only, this was analyzed in [19]. There it was concluded that higher loop corrections are present, but can in fact be absorbed into a field redefinition, or equivalently, by a coordinate transformation on the HM moduli space. In other words, if the appropriate variables are chosen, there are no additional corrections beyond one-loop. We will argue in this section that such a non-renormalization theorem might also work in the generic case including an arbitrary number of hypermultiplets.

Non-renormalization theorems are best understood in an off-shell formulation. We have seen that at one-loop order, the superspace lagrangian is given by (4.5) and that η^0 is the dilaton multiplet, at least in the chosen gauge $v^0 = \bar{v}^0 = 0$ for which $x^0 = \mathrm{e}^{-\phi} = g_s^2$. In this gauge, there is a natural generalization to higher loop corrections,

$$\mathcal{F}(v, \bar{v}, x) = -\mathrm{Im} \sum_{n=0}^{\infty} \left[\oint_{\mathcal{C}_0} \frac{\mathrm{d}\zeta}{2\pi i \zeta} (x^0)^{n-1} F_n(\eta^\Lambda) \right] , \tag{5.1}$$

where $F_0(\eta^\Lambda)$ and $F_2(\eta^\Lambda)$ are given in (4.5) and the contour is again chosen around the origin. Furthermore, as explained in the beginning of the previous section, we have that $F_1 = 0$. The F_n with $n \geq 3$ are coefficient functions defining possible higher loop corrections. The homogeneity condition needed for superconformal invariance requires F_n to be homogeneous of degree $2-n$. If needed, logarithms of x^0 could be included in F_n, as is the case e.g. for $n = 2$, as long as the homogeneity conditions are satisfied under the contour integral.

It is now easy to see that all terms with $n \geq 3$ vanish under the contour integral. Indeed, using the homogeneity properties of the F_n, (5.1) can be rewritten as

$$\mathcal{F}(v, \bar{v}, x) = -\mathrm{Im} \sum_{n=0}^{\infty} \left[(x^0)^{n-1} \oint \frac{\mathrm{d}\zeta}{2\pi i} \, \zeta^{n-3} F_n(\zeta \eta^\Lambda) \right] , \tag{5.2}$$

with, as usual, $\zeta\eta^\Lambda = v^\Lambda + x^\Lambda \zeta - \bar{v}^\Lambda \zeta^2$, and the contour is chosen around the origin. If we now assume that the $F_n(\zeta\eta^\Lambda)$ have no singularities around the origin in the ζ plane, then it is clear that all terms with $n \geq 3$ vanish under the contour integral. This amounts to a non-renormalization theorem for hypermultiplets (or, equivalently, off-shell tensor multiplets).

The following important remark is in order. Notice that we have assumed the contour to be around the origin. This choice is intimately related to the choice of SU(2) gauge, and the identification $x^0 = \mathrm{e}^{-\phi}$. We could relax this assumption. If different contours are taken, it might be that they enclose poles of F_n away from the origin, which could yield non-vanishing contributions to the contour integral. In fact, one should consider the more SU(2) covariant expression

$$\mathcal{F}(v, \bar{v}, x) = -\mathrm{Im} \sum_{n=0}^{\infty} \left[\oint_{\mathcal{C}} \frac{\mathrm{d}\zeta}{2\pi i \zeta} (\eta^0)^{n-1} F_n(\eta^\Lambda) \right] , \tag{5.3}$$

where the contour encloses all poles. These would yield non-vanishing higher-loop corrections. In fact, we know from [18] and [19] that such corrections can appear. It might

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however be that these higher-loop corrections can be absorbed by redefining the dilaton. In our language, this would correspond to a modification of the identification between x^0 and $\mathrm{e}^{-\phi}$.

In the gauge $v^0 = 0$, (5.3) reduces to (5.1), in which the absense of higher-loop corrections is manifest, provided the contour is taken to be around the origin. This choice of contour works for sure at one-loop order. To fully prove the non-renormalization conjecture, a better understanding of how to choose the contours would be needed. We leave this for future investigation.

6. Discussion and outlook

In this paper we combined superspace and conformal calculus techniques to determine the string loop corrections to the hypermultiplet moduli space of type II string theory compactified on a generic Calabi-Yau threefold. The resulting lagrangian is completely specified by a single function $H(\eta^I)$ homogeneous of degree one. The result of our analysis is that this function is given by

$$H^{\mathrm{qc}}(\eta^I) = \frac{1}{\eta^0}\, F(\eta^\Lambda) + \frac{\chi}{3\pi}\, D(\eta^\Lambda)\, \eta^0 \ln(\eta^0)\ . \tag{6.1}$$

The η-symbols are $n_H + 1$ $\mathcal{N} = 2$ tensor multiplets which include the dilaton multiplet. The first term corresponds to the classical term and was obtained in [26]. Here $F(\eta^\Lambda)$ is homogeneous of degree two, and arises from the vector multiplets after doing the c-map. The second term in (6.1) is proportional to the Euler number of the Calabi-Yau χ and describes the one-loop corrections. It contains a function $D(\eta)$, homogeneous of degree zero. To match with known type II string amplitudes this function should be taken constant:

$$D_{\mathrm{IIA}} = -i\ , \qquad D_{\mathrm{IIB}} = i\ . \tag{6.2}$$

We furthermore argued that there is a non-renormalization theorem, excluding possible higher loop corrections. It is fair to say that this is a conjecture rather than a theorem.

It is somewhat intriguing that our generic effective action allows for a non-constant holomorphic function $D(z)$, homogeneous of degree zero. At present, no string amplitude at one-loop seems to contribute to $D(z)$ apart from the constant term. It would be interesting to know if this is an exact result to all orders in α'.

It remains an open problem to determine the non-perturbative corrections to the hypermultiplet moduli space which arise from Euclidean D-branes wrapping supersymmetric cycles in the Calabi-Yau threefold [14]. These non-perturbative corrections are expected to break the Peccei-Quinn symmetries, making a description in terms of tensor multiplets more difficult, unless one takes into account global issues in the scalar-tensor duality (see e.g. [46, 15]).

A less ambitious extension of our present work is to consider the gauging of the isometry group of our perturbatively corrected hypermultiplet moduli spaces[11] along the lines [48]

[11] For the perturbatively corrected universal hypermultiplet such an investigation was performed in [47, 16].

which corresponds to considering Calabi-Yau compactifications with non-trivial background fluxes. Based on the observations in orientifold compactifications of the type II string to $\mathcal{N} = 1$ supergravity [3, 49] one would expect that perturbative corrections can play an important role in altering the vacuum structure and stabilizing the moduli of the compactification. In this context it would also be interesting to consider the orientifold projection of our string loop corrected $\mathcal{N} = 2$ supergravity along the lines [50 − 52]. This could lead to results complementary to the ones obtained in [53] where perturbative corrections to the Kähler potential describing the vector multiplet sector have been studied.

Finally let us remark that based on the string-string duality [54] between the type IIA string compactified on Calabi-Yau threefolds and the heterotic string on $K3 \times T^2$, we expect that our results are also valid for heterotic string compactifications. In this context it would certainly be interesting to understand how the string loop corrected hypermultiplet moduli spaces found in this paper arise in the dual heterotic picture, since there the corresponding moduli space is exact in the string coupling constant.

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A. Derivation of the loop-corrected lagrangians

This appendix collects the technical details for deriving the quantum-corrected hyper- and tensor multiplet lagrangians presented in section 4. The calculation follows the template outlined in subsection 3.1 and naturally splits into the three parts of determining the superconformal gauge fixing conditions, constructing the tensor multiplet lagrangian, and dualizing the tensor into hypermultiplets. These steps will be carried out in subsections A.1, A.2 and A.3, respectively. As an example, we will discuss the quantum corrected universal hypermultiplet in subsection A.4. Note that the construction of the classical results given in subsection 3.2 is completely analogous to the quantum case, so that its intermediate steps can simply be obtained from the quantum formulae by switching off the deformation piece by setting $c = D(v) = \Delta = 0$.

A.1 Gauge fixing the superconformal symmetries

The starting point of our construction is the function (4.6)

$$\mathcal{F}(x^0, v^\Lambda, \bar{v}^\Lambda, x^\Lambda) = \frac{1}{4x^0}\left(N_{\Lambda\Sigma}x^\Lambda x^\Sigma - 2K(v,\bar{v})\right) - 2ic\left[D(v^\Lambda) - \bar{D}(\bar{v}^\Lambda)\right]\, x^0 \,\ln(x^0)\,, \quad \text{(A.1)}$$

which is obtained by evaluating the contour integral of $H^{\rm qc}(\eta)$, eq. (4.5). We then compute the second derivatives of $\mathcal{F}$ which enter into the gauge-fixed lagrangian (3.11). Adopting the SU(2) gauge-choice (3.6) where $v^0 = 0, v^1 = \bar{v}^1$ and in terms of the inhomogeneous coordinates (3.14) these derivatives read

$$\begin{aligned}
\mathcal{F}_{x^0x^0} &= \frac{1}{x^0}\left[2N_{\Lambda\Sigma}\,A^\Lambda\,A^\Sigma - \frac{(v^1)^2}{(x^0)^2}\,K(z,\bar{z}) - 4\,\Delta\right] ,\\
\mathcal{F}_{x^0v^\Lambda} &= \frac{v^1}{2(x^0)^2}\bar{z}^\Sigma N_{\Lambda\Sigma} - \frac{1}{v^1}(\partial_\Lambda N_{\Sigma\Xi})\,A^\Sigma\,A^\Xi - \frac{2ic}{v^1}\left(1+\ln(x^0)\right)(\partial_\Lambda D)\ ,\\
\mathcal{F}_{x^0x^\Lambda} &= -\ \frac{1}{x^0}\,N_{\Lambda\Sigma}\,A^\Sigma\,,\\
\mathcal{F}_{x^\Lambda v^\Sigma} &= \frac{1}{v^1}(\partial_\Lambda N_{\Sigma\Xi})\,A^\Xi\,,\\
\mathcal{F}_{x^\Lambda x^\Sigma} &= \frac{1}{2(x^0)}N_{\Lambda\Sigma}\,.
\end{aligned} \tag{A.2}$$

Here $\partial_\Lambda \equiv \partial/\partial v^\Lambda$ and we have used the homogeneity properties of $K(v,\bar{v})$, $N_{\Lambda\Sigma}$, $D(v)$, $\partial_\Lambda N_{\Sigma\Xi}$, and $\partial_\Lambda D(v)$ to write these as functions of $z,\bar{z}$ by extracting appropriate powers of v^1. Furthermore, we defined

$$\Delta \equiv \frac{ic}{2}\left[D(z) - \bar{D}(\bar{z})\right], \tag{A.3}$$

and we will use $K \equiv K(z,\bar{z}) = z^\Lambda N_{\Lambda\Sigma}\bar{z}^\Sigma$ from now on.

In order to fix the remaining superconformal symmetry we still need to implement the embedding equation (3.10). Setting $\kappa^2 = 1/2$ and substituting the derivatives (A.2), eq. (3.10) is easily solved for v^1:

$$v^1 = \sqrt{\frac{4x^0(1+\Delta x^0)}{K}}\,. \tag{A.4}$$

This relation expresses v^1 in terms of $x^0, x^\Lambda, z^a, \bar{z}^a$. Together with A^Λ, these provide the coordinates on the manifold $\mathcal{M}_{\rm TM}$. The SU(2)-gauge (3.6), eq. (A.4) fixes the superconformal transformations SU(2) and dilatations.

A.2 Constructing the tensor multiplet lagrangian

Our next task is to compute the SU(2) currents (3.3) and (3.4) together with the inverse of the matrix M, eq. (3.5). The former are obtained by substituting the derivatives (A.2) into the general expressions for $\vec{S}_\mu$ and $\vec{T}_\mu$ and read

$$\vec{S}_\mu = \begin{bmatrix} -i\left(\frac{x^0(1+\Delta x^0)}{K}\right)^{1/2} z^\Lambda N_{\Lambda\Sigma}\partial_\mu A^\Sigma \\ i\left(\frac{x^0(1+\Delta x^0)}{K}\right)^{1/2} \bar{z}^\Lambda N_{\Lambda\Sigma}\partial_\mu A^\Sigma \\ -2i\frac{(1+\Delta x^0)}{K}\left(\partial_a K\partial_\mu z^a - \partial_{\bar{a}}K\partial_\mu \bar{z}^{\bar{a}}\right) \end{bmatrix}, \tag{A.5}$$

and

$$\vec{T}_\mu = \begin{bmatrix} \left(\frac{1+\Delta x^0}{x^0 K}\right)^{1/2}\left(z^\Lambda N_{\Lambda\Sigma}E^\Sigma_\mu - 2z^\Lambda N_{\Lambda\Sigma}A^\Sigma E^0_\mu\right) \\ \left(\frac{1+\Delta x^0}{x^0 K}\right)^{1/2}\left(\bar{z}^\Lambda N_{\Lambda\Sigma}E^\Sigma_\mu - 2\bar{z}^\Lambda N_{\Lambda\Sigma}A^\Sigma E^0_\mu\right) \\ -\frac{4}{x^0}(1+2\Delta x^0)E^0_\mu \end{bmatrix}, \tag{A.6}$$

respectively. Computing the matrix M, eq. (3.5), we find that it is block-diagonal with entries $M_{\pm 3} = 0$. The determinant of the 2×2-block is denoted by

$$|\tilde{M}| = \frac{(1+\Delta x^0)^2}{K^2}(zNz)(\bar{z}N\bar{z}) - \Delta^2 (x^0)^2\,, \tag{A.7}$$

and it is easy to show that this coincides with (4.12). The matrix M^{-1} is found as

$$M^{-1} = \begin{bmatrix} -\frac{(1+\Delta x^0)}{|\tilde{M}|K}(\bar{z}N\bar{z}) & \frac{\Delta x^0}{|\tilde{M}|} & 0 \\ \frac{\Delta x^0}{|\tilde{M}|} & -\frac{(1+\Delta x^0)}{|\tilde{M}|K}(zNz) & 0 \\ 0 & 0 & \frac{1}{4(1+\Delta x^0)} \end{bmatrix}. \tag{A.8}$$

Here $zNz = z^\Lambda N_{\Lambda\Sigma} z^\Sigma$, etc.

The next step then consists in substituting these results into the lagrangian (3.11) and employing the gauge fixing conditions (3.10) and (A.4). In order to make this calculation more traceable, we take advantage of the fact that the gauge-fixed lagrangian naturally splits into three sectors where the two space-time derivatives act on scalar-scalar (SS), tensor-tensor (TT) and scalar-tensor (ST) fields, respectively. We will now consider these sectors in turn.

A.2.1 The scalar-scalar sector

The part of the lagrangian (3.11) where the two space-time derivatives act on scalar fields is given by

$$e^{-1}\mathcal{L}^{\rm SS} = \mathcal{F}_{x^I x^J}\left(\partial_\mu v^I \partial^\mu \bar{v}^J + \frac{1}{4}\partial_\mu x^I \partial^\mu x^J\right) - \frac{1}{2}\vec{S}_\mu M^{-1}\vec{S}^\mu\,. \tag{A.9}$$

Substituting M^{-1} from eq. (A.8) and $\vec{S}_\mu$ given in (A.5), this becomes

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm SS}_{\rm qc} = &- \frac{1+2\Delta x^0}{2(x^0)^2\,(1+\Delta x^0)}\,(\partial_\mu x^0)^2 + 2(1+\Delta x^0)\mathcal{G}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} \\ &+ 2\,x^0\, g_{\Lambda\Sigma}\,\partial_\mu A^\Lambda \partial^\mu A^\Sigma - \frac{c^2(x^0)^2}{8(1+\Delta x^0)}\left(\partial_a D \partial_\mu z^a - \partial_{\bar{a}}\bar{D}\partial_\mu \bar{z}^{\bar{a}}\right)^2\,. \end{aligned} \tag{A.10}$$

Here $g_{\Lambda\Sigma}$ is given by

$$\begin{aligned} g_{\Lambda\Sigma} = &\frac{1}{4}\left[N_{\Lambda\Sigma} - \frac{(1+\Delta x^0)^2}{|\tilde{M}|K^2}\left[(zNz)(N\bar{z})_\Lambda (N\bar{z})_\Sigma + (\bar{z}N\bar{z})(Nz)_\Lambda (Nz)_\Sigma\right]\right] \\ &- \left(\frac{\Delta}{2}\right)\left(\frac{x^0\,(1+\Delta x^0)}{|\tilde{M}|K}\right)(Nz)_\Lambda (N\bar{z})_\Sigma\,, \end{aligned} \tag{A.11}$$

and the matrix $\mathcal{G}_{a\bar{b}}$, defined in (2.2), appears as a subsector of[12]

$$\mathcal{G}_{\Lambda\bar{\Sigma}} = \partial_\Lambda \partial_{\bar{\Sigma}} \ln K \;=\; \frac{1}{K}\left[N_{\Lambda\bar{\Sigma}} - \frac{1}{K}(Nz)_\Lambda (N\bar{z})_{\bar{\Sigma}}\right]. \tag{A.12}$$

[12]This is completely analogous to taking the superconformal quotient for vector multiplets [5] where the choice $z^\Lambda = v^\Lambda / v^1$ corresponds to introducing special coordinates.

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We then construct the matrix

$$\mathcal{M}_{\Lambda\Sigma} = -i\bar{F}_{\Lambda\Sigma} - \frac{(1+\Delta x^0)}{|\tilde{M}|K}\left[\frac{(1+\Delta x^0)}{K}(\bar{z}N\bar{z})(Nz)_\Lambda(Nz)_\Sigma + (\Delta x^0)(Nz)_\Lambda(N\bar{z})_\Sigma\right] . \tag{A.13}$$

It can be rewritten as in (4.10) and satisfies the relation

$$g_{\Lambda\Sigma} = \frac{1}{4}\left(\mathcal{M}+\bar{\mathcal{M}}\right)_{\Lambda\Sigma} . \tag{A.14}$$

This decomposition is motivated by the classical limit where, upon setting $c = 0$, $\mathcal{M}_{\Lambda\Sigma}$ reduces to $\mathcal{N}_{\Lambda\Sigma}$ given in (2.3). This implies that $g_{\Lambda\Sigma}|_{c=0} = \frac{1}{4}\left(\mathcal{N}+\bar{\mathcal{N}}\right)_{\Lambda\Sigma}$ precisely reproduces the scalar kinetic term of $(\partial_\mu A)^2$ in the classical tensor multiplet lagrangian (3.16).

Using (A.14) and the definition $\tilde{A}_\mu = i\left[\partial_a D(z)\partial_\mu z^a - \partial_{\bar{a}}\bar{D}(\bar{z})\partial_\mu \bar{z}^{\bar{a}}\right]$ given in (4.16) the lagrangian (A.10) becomes

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm SS}_{\rm qc} = &-\frac{1+2\Delta x^0}{2(x^0)^2\,(1+\Delta x^0)}\,(\partial_\mu x^0)^2 + 2(1+\Delta x^0)\mathcal{G}_{a\bar{b}}\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}} \\ &+\frac{1}{2}\,x^0\,\left(\mathcal{M}+\bar{\mathcal{M}}\right)_{\Lambda\Sigma}\,\partial_\mu A^\Lambda\partial^\mu A^\Sigma + \frac{c^2(x^0)^2}{8(1+\Delta x^0)}\tilde{A}_\mu\,\tilde{A}^\mu . \end{aligned} \tag{A.15}$$

A.2.2 The tensor-tensor sector

The part of the TM lagrangian (3.11) where the two space-time derivatives act on tensor fields reads

$$e^{-1}\mathcal{L}^{\rm TT} = -\,\mathcal{F}_{x^I x^J}\,E^I_\mu\,E^{J\mu} - \frac{1}{2}\,\vec{T}_\mu\,(M^{-1})\,\vec{T}^\mu . \tag{A.16}$$

Substituting the derivatives (A.2) together with M^{-1} and $\vec{T}_\mu$ given above it is straightforward to rewrite this expression as

$$e^{-1}\mathcal{L}^{\rm TT} = 2\,\mathcal{T}^{\rm qc}_{IJ}\,E^I_\mu\,E^{J\mu} , \tag{A.17}$$

with $\mathcal{T}^{\rm qc}_{IJ}$ being the symmetric matrix

$$\mathcal{T}^{\rm qc}_{IJ} = \begin{bmatrix} \frac{1+2\Delta x^0}{(x^0)^2(1+\Delta x^0)} + 4\mathcal{T}^{\rm qc}_{\Lambda\Sigma}\,A^\Lambda\,A^\Sigma & -2\,\mathcal{T}^{\rm qc}_{\Lambda\Sigma}\,A^\Sigma \\ -2\,A^\Lambda\,\mathcal{T}^{\rm qc}_{\Lambda\Sigma} & \mathcal{T}^{\rm qc}_{\Lambda\Sigma} \end{bmatrix} , \tag{A.18}$$

and

$$\begin{aligned} \mathcal{T}^{\rm qc}_{\Lambda\Sigma} = &-\frac{1}{4x^0}\left[N_{\Lambda\Sigma} - \frac{(1+\Delta x^0)^2}{|\tilde{M}|K^2}\Big[(zNz)(N\bar{z})_\Lambda(N\bar{z})_\Sigma + (\bar{z}N\bar{z})(Nz)_\Lambda(Nz)_\Sigma\Big]\right] \\ &-\left(\frac{\Delta}{4}\right)\frac{1+\Delta x^0}{|\tilde{M}|K}\left[(Nz)_\Lambda(N\bar{z})_\Sigma + (N\bar{z})_\Lambda(Nz)_\Sigma\right] . \end{aligned} \tag{A.19}$$

This expression can be rewritten as (4.14) in the main text.

A.2.3 The scalar-tensor sector

Finally, we have the sector where the two space-time derivatives act on a scalar and a tensor field. In this subsector eq. (3.11) gives rise to

$$e^{-1}\mathcal{L}^{\rm ST} = iE^I_\mu(\mathcal{F}_{v^I x^J}\partial_\mu v^I - \mathcal{F}_{\bar v^I x^J}\partial^\mu \bar v^J) - \vec S_\mu(M^{-1})\vec T^\mu \,. \tag{A.20}$$

Substituting the results (A.8), (A.5) and (A.6) this becomes

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm ST} = \; & \partial_\mu\left(F_{\Lambda\Sigma}+\bar F_{\Lambda\Sigma}\right)\left(A^\Lambda A^\Sigma E^{0\mu} - A^\Lambda\, E^{\Sigma\mu}\right) \\ & -\frac{i(1+\Delta x^0)^2}{|\tilde M|K^2}\left[(zNz)(N\bar z)_\Lambda (N\bar z)_\Sigma - \text{c.c.}\right]\left(2A^\Lambda\partial_\mu A^\Sigma E^{0\mu} - E^\Lambda_\mu \partial^\mu A^\Sigma\right) \\ & + i\Delta x^0\,\frac{(1+\Delta x^0)}{K}\left[(Nz)_\Lambda (N\bar z)_\Sigma - (N\bar z)_\Lambda (Nz)_\Sigma\right]\left(2A^\Lambda\partial_\mu A^\Sigma E^{0\mu} - E^\Lambda_\mu \partial^\mu A^\Sigma\right) \\ & -\frac{2i\Delta}{K}\left[(N\bar z)_\Lambda \partial^\mu z^\Lambda - (Nz)_\Lambda \partial^\mu \bar z^\Lambda\right] E^0_\mu - \frac{2c}{x^0}\left(D(z)+\bar D(\bar z)\right) E^0_\mu \partial^\mu x^0 \,. \end{aligned} \tag{A.21}$$

Here we have used the homogeneity property of $F_{\Lambda\Sigma}$ to rewrite space-time derivatives acting on $z^\Lambda, \bar z^\Lambda$ as space-time derivatives of $F_{\Lambda\Sigma}$ in the first line. Furthermore, the last term was integrated by parts making use of the Bianci identity for the tensor field strength, $\partial^\mu E^I_\mu = 0$.

Partially integrating the first term in (A.21), we find that the first three lines are all proportional to the combination $\left(2A^\Lambda\partial_\mu A^\Sigma E^{0\mu} - E^\Lambda_\mu\partial^\mu A^\Sigma\right)$. The prefactor of this term can then be reexpressed in terms of the matrix $\mathcal{M}_{\Lambda\Sigma}$ defined in (4.10) by noting that

$$\begin{aligned} i\left(\mathcal{M}-\bar{\mathcal{M}}\right)_{\Lambda\Sigma} = & F_{\Lambda\Sigma}+\bar F_{\Lambda\Sigma} \\ & + i\frac{(1+\Delta x^0)^2}{|\tilde M|K^2}\left[(zNz)(N\bar z)_\Lambda (N\bar z)_\Sigma - (\bar z N\bar z)(Nz)_\Lambda (Nz)_\Sigma\,\right] \\ & - i\Delta x^0\frac{(1+\Delta x^0)}{K}\left[(Nz)_\Lambda (N\bar z)_\Sigma - (N\bar z)_\Lambda (Nz)_\Sigma\right] . \end{aligned} \tag{A.22}$$

Recalling the definition of the Kähler connection from eq. (4.10) the expression (A.21) can then be concisely written as

$$e^{-1}\mathcal{L}^{\rm ST}_{\rm qc} = i\left(\mathcal{M}-\bar{\mathcal{M}}\right)_{\Lambda\Sigma}\left[E^\Lambda_\mu\partial^\mu A^\Sigma - 2E^0_\mu A^\Lambda \partial^\mu A^\Sigma\right] - 2\Delta A_\mu E^{0\mu} - \frac{2c}{x^0}\left(D+\bar D\right)E^0_\mu\partial^\mu x^0 \,. \tag{A.23}$$

Summing the scalar-scalar (A.15), tensor-tensor (A.17), and scalar-tensor contributions (A.23) then yields the deformed tensor multiplet lagrangian

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm TM}_{\rm qc} = & -\frac{1+2\Delta x^0}{2(x^0)^2\,(1+\Delta x^0)}\,(\partial_\mu x^0)^2 + 2(1+\Delta x^0)\mathcal{G}_{a\bar b}\partial_\mu z^a \partial^\mu \bar z^{\bar b} \\ & +\frac{1}{2}\,x^0\,\left(\mathcal{M}+\bar{\mathcal{M}}\right)_{\Lambda\Sigma}\,\partial_\mu A^\Lambda \partial^\mu A^\Sigma + \frac{c^2(x^0)^2}{8(1+\Delta x^0)}\tilde A_\mu\,\tilde A^\mu + 2\,\mathcal{T}^{\rm qc}_{IJ}\,E^I_\mu\,E^{J\mu} \\ & + i(\mathcal{M}-\bar{\mathcal{M}})_{\Lambda\Sigma}\left[E^\Lambda_\mu\partial^\mu A^\Sigma - 2E^0_\mu A^\Lambda\partial^\mu A^\Sigma\right] - 2\Delta A_\mu E^{0\mu} - \frac{2c}{x^0}\left(D+\bar D\right)E^0_\mu\partial^\mu x^0 . \end{aligned} \tag{A.24}$$

Here $\mathcal{T}^{\rm qc}_{IJ}$ is given in eq. (A.18). Carrying out the coordinate transformation $x^0 = {\rm e}^{-\phi}$ this expression then gives rise to the deformed TM lagrangian (4.8). This result then completes the derivation of the results presented in section 4.2.

We end this subsection by taking the classical limit of (A.24) by setting $c = \Delta = 0$. In this case we have

$$\mathcal{M}_{\Lambda\Sigma}|_{c=0} = \mathcal{N}_{\Lambda\Sigma} \tag{A.25}$$

and the matrix appearing in the tensor kinetic term simplifies to

$$\mathcal{T}^{\rm cl}_{IJ} = \frac{1}{x^0} \begin{bmatrix} \frac{1}{x^0} - \left(\mathcal{N}+\bar{\mathcal{N}}\right)_{\Lambda\Sigma} A^\Lambda A^\Sigma & \frac{1}{2} A^\Lambda \left(\mathcal{N}+\bar{\mathcal{N}}\right)_{\Lambda\Sigma} \\ \frac{1}{2} \left(\mathcal{N}+\bar{\mathcal{N}}\right)_{\Lambda\Sigma} A^\Sigma & -\frac{1}{4}\left(\mathcal{N}+\bar{\mathcal{N}}\right)_{\Lambda\Sigma} \end{bmatrix} . \tag{A.26}$$

The resulting classical tensor multiplet lagrangian is then precisely given by (3.16).

A.3 Dualizing to hypermultiplets

After establishing the deformed TM lagrangian (3.16) we can now construct the quantum corrected HM lagrangian by dualizing the tensor fields into scalars. For this purpose we add the Lagrange multipliers introduced in eq. (3.19),

$$e^{-1}\mathcal{L}^{\rm LM} = 2(\partial_\mu w_0) E^{0\mu} - (\partial_\mu B_\Lambda) E^{\Lambda\mu} , \tag{A.27}$$

to the deformed TM lagrangian and compute the equations of motion for the tensor fields. Using the notation $E^I_\mu = \{E^0_\mu, E^\Lambda_\mu\}$, we find

$$E^I_\mu = -\frac{1}{4} \left(\mathcal{T}^{\rm qc}\right)^{IJ} \mathcal{J}_{J\mu} . \tag{A.28}$$

Here

$$\left(\mathcal{T}^{\rm qc}\right)^{IJ} = \frac{(x^0)^2}{f} \begin{bmatrix} 1 & 2A^\Sigma \\ 2A^\Lambda & \frac{f}{(x^0)^2}(\mathcal{T}^{\rm qc})^{\Lambda\Sigma} + 4A^\Lambda A^\Sigma \end{bmatrix} , \quad f = \frac{1+2\Delta x^0}{1+\Delta x^0} , \tag{A.29}$$

and $(\mathcal{T}^{\rm qc})^{\Lambda\Sigma}$ denotes the inverse of (A.19). Furthermore, we have introduced the currents coupling to the tensor field strength $E^{\Lambda\mu}$ and $E^{0\mu}$

$$\begin{aligned} \mathcal{J}_{\Lambda\mu} &= -\left(\partial_\mu B_\Lambda\right) + i(\mathcal{M}-\bar{\mathcal{M}})_{\Lambda\Sigma}\partial_\mu A^\Sigma , \quad \text{and} \\ \mathcal{J}_{0\mu} &= 2(\partial_\mu w_0) - 2i\,(\mathcal{M}-\bar{\mathcal{M}})_{\Lambda\Sigma} A^\Lambda \partial_\mu A^\Sigma - 2\Delta A_\mu - \frac{2c}{x^0}(D(z)+\bar{D}(\bar{z}))(\partial_\mu x^0) , \end{aligned} \tag{A.30}$$

respectively. The dual deformed hypermultiplet lagrangian is then obtained by using the equations of motion (A.28) to eliminate the tensor fields from $\mathcal{L}^{\rm TM}_{\rm qc} + \mathcal{L}^{\rm LM}$. It reads

$$\begin{aligned} e^{-1}\mathcal{L}^{\rm HM}_{\rm qc} = & -\frac{1+2\Delta x^0}{2(x^0)^2(1+\Delta x^0)}(\partial_\mu x^0)^2 + 2(1+\Delta x^0)\mathcal{G}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} \\ & -\frac{1}{8}(\mathcal{T}^{\rm qc})^{IJ}\mathcal{J}_{I\mu}\mathcal{J}^\mu_J + \frac{x^0}{2}(\mathcal{M}+\bar{\mathcal{M}})_{\Lambda\Sigma}\partial_\mu A^\Lambda \partial^\mu A^\Sigma + \frac{c^2(x^0)^2}{8(1+\Delta x^0)}\tilde{A}_\mu \tilde{A}^\mu . \end{aligned} \tag{A.31}$$

Upon expanding the terms containing $\mathcal{J}^\mu_I$ using the definitions (A.30) and setting $x^0 = {\rm e}^{-\phi}$ this expression gives rise to the deformed hypermultiplet lagrangian (4.17). Note that in the classical limit $c = 0$ (A.31) reduces to the classical hypermultiplet lagrangian obtained from the c-map (2.4) with, as usual, w_0 given by (3.21).

A.4 The universal hypermultiplet example

In order to wrap up this section we now apply the formalism outlined in the previous subsections to the universal hypermultiplet. This HM geometry arises from compactifying type IIA strings on a rigid CY_3 ($h^{1,2} = 0$) and consists of a single (the universal) hypermultiplet. Classically this hypermultiplet parameterizes the manifold

$$\mathcal{M}_{\mathrm{UHM}} = \frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2) \times \mathrm{U}(1)}\,, \tag{A.32}$$

and the string loop corrections to this space have recently been found in [19]. It is thus interesting to compare our results for generic CY_3 compactifications to the ones obtained in the rigid limit.

In our conventions, the special Kähler geometry underlying the classical universal hypermultiplet is determined by the holomorphic prepotential

$$F^{\mathrm{UHM}}(v) = -i(v^1)^2\,. \tag{A.33}$$

Furthermore, since rigid CY_3 do not have any complex structure moduli, the function $D(v)$ appearing in the loop-correction term has to be an imaginary constant. Without loss of generality we can set $D(v) = -i$ since any rescaling of $D(v)$ can be absorbed into a rescaling of the constant c.

Starting from the prepotential (A.33) it is straightforward to calculate the objects specifying the corresponding special Kähler geometry (2.8)

$$N_{11} = 4\,, \quad K(v,\bar{v}) = 4v^1\bar{v}^1\,, \quad K(z,\bar{z}) = 4\,. \tag{A.34}$$

Here we used the definition of the inhomogeneous coordinates (3.14) to obtain the last expression. Substituting these quantities into the general formula for $\mathcal{F}(x,v,\bar{v})$, eq. (4.6), we obtain

$$\mathcal{F}^{\mathrm{UHM}}(v,\bar{v},x) = \frac{1}{x^0}\,\left((x^1)^2 - 2\,v^1\,\bar{v}^1\,\right) - 4\,c\,x^0\,\ln(x^0)\,. \tag{A.35}$$

Note that this result agrees with the one obtained from evaluating the contour integral for $H^{\mathrm{UHM}}(\eta)$, eq. (2.13), in the partial gauge $v^0 = 0$ and for a contour encircling the origin.

In order to obtain the loop-corrected tensor multiplet lagrangian corresponding to the universal hypermultiplet we can then proceed by evaluating the definitions (4.14) and (4.10) for the specific prepotential (A.33). Using that (A.7) becomes

$$|\tilde{M}| = 1 + 2cx^0\,, \tag{A.36}$$

these read

$$\mathcal{T}^{\mathrm{qc}}_{11} = \frac{1}{x^0\,(1+2cx^0)}\,, \quad \mathcal{M}_{11} = -2\left(1+2cx^0\right)\,. \tag{A.37}$$

Furthermore, the connections (4.16) and (4.15) vanish in the absence of complex structure moduli.

Substituting these results into the general lagrangian (3.11) we obtain a double tensor multiplet description of the quantum corrected universal hypermultiplet[13]

$$e^{-1}\mathcal{L}_{\rm qc}^{\rm DTM} = -\frac{1}{2(x^0)^2}\,\frac{1+2cx^0}{1+cx^0}(\partial_\mu x^0)^2 + 2x^0\left(1+2cx^0\right)(\partial_\mu A)^2 + 2\,\mathcal{T}_{IJ}^{\rm DTM}\,E_\mu^I\,E^{J\mu}\,, \quad (A.38)$$

where $\mathcal{T}_{IJ}^{\rm DTM}$ is given by

$$\mathcal{T}_{IJ}^{\rm DTM} = \frac{1}{x^0(1+2cx^0)}\begin{bmatrix} \left(\frac{1}{x^0}\,\frac{(1+2cx^0)^2}{1+cx^0} + 4\,A^2\right) & -2\,A \\ -\,2A & 1 \end{bmatrix}. \quad (A.39)$$

and we have dropped the (redundant) index from A^1.

We note that taking the classical limit $c = 0$ and using the coordinate transformation

$$x^0 = \mathrm{e}^{-\phi}\,, \quad A = \frac{1}{2}\chi\,, \quad E^{I\mu} = \frac{1}{2}H^{\mu I}\,, \quad (A.40)$$

this is precisely the double tensor multiplet lagrangian obtained in ref. [20]

$$e^{-1}\mathcal{L}^{\rm DTM} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\mathrm{e}^{-\phi}\partial^\mu\chi\partial_\mu\chi + \frac{1}{2}M_{IJ}H^{\mu I}H_\mu^J \quad (A.41)$$

where

$$M_{IJ} = \mathrm{e}^{\phi}\begin{bmatrix} \mathrm{e}^{\phi}+\chi^2 & -\chi \\ -\chi & 1 \end{bmatrix}. \quad (A.42)$$

In order to obtain the quantum corrected HM lagrangian, we substitute the equations (A.37) into (4.17). Denoting $f = \frac{1+2cx^0}{1+cx^0}$ as in eq. (4.10) the result becomes

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm qc}^{\rm UHM} = &-\frac{f}{2(x^0)^2}(\partial_\mu x^0)^2 - \frac{1}{2}x^0\left(1+2cx^0\right)\left(4(\partial_\mu A)^2 + \frac{1}{4}(\partial_\mu B)^2\right) \\ &- \frac{(x^0)^2}{2f}\left(\partial_\mu w_0 - A\partial_\mu B\right)^2 . \end{aligned} \quad (A.43)$$

One can easily check (using Mathematica) that the metric of this non-linear sigma model is Einstein with Ricci curvature $R = -6$.

We now compare the lagrangian (A.43) to the result for the loop corrected universal hypermultiplet moduli space found in [19]. There the corresponding line element was given in Calderbank-Pedersen form [55] as:

$$\frac{1}{2}\,\mathrm{d}s^2 = \frac{(\rho^2+\hat{\chi})}{(\rho^2-\hat{\chi})^2}\left[(\mathrm{d}\rho)^2 + (\mathrm{d}\eta)^2 + \frac{(\mathrm{d}\phi)^2}{4}\right] + \frac{\rho^2}{(\rho^2-\hat{\chi})^2(\rho^2+\hat{\chi})}\left[\mathrm{d}\psi + \eta\,\mathrm{d}\phi\right]^2 . \quad (A.44)$$

Moreover, the value of the constant $\hat{\chi}$ was determined to be

$$\hat{\chi} \equiv -\frac{4\,\zeta(2)\,\chi(X)}{(2\pi)^3} = -\frac{1}{6\pi}\left(h^{1,1} - h^{1,2}\right). \quad (A.45)$$

[13] Since the universal hypermultiplet possesses different and inequivalent pairs of commuting isometries which can be used to dualize scalars into tensor fields, the description of the universal hypermultiplet in terms of a double tensor multiplet is not unique [24], so strictly speaking a “universal double tensor multiplet” does not exist.

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In order to match (A.43) with (A.44) we perform the coordinate transformation

$$x^0 = \left(\rho^2 - c\right)^{-1} \,, \quad A = \eta \,, \quad B = 2\phi \,, \quad w_0 = -2\psi \,, \tag{A.46}$$

which brings (A.43) into Calderbank-Pedersen form:[14]

$$\frac{1}{2}\,\mathrm{d}s^2 = \frac{(\rho^2 + c)}{(\rho^2 - c)^2}\left[(\mathrm{d}\rho)^2 + (\mathrm{d}\eta)^2 + \frac{(\mathrm{d}\phi)^2}{4}\right] + \frac{\rho^2}{(\rho^2 - c)^2(\rho^2 + c)}\left[\mathrm{d}\psi + \eta\,\mathrm{d}\phi\right]^2 \,. \tag{A.47}$$

Comparing the expressions (A.44) and (A.47) allows us to read off the value of the constant c as

$$c = \hat{\chi} \equiv -\frac{4\,\zeta(2)\,\chi(X)}{(2\pi)^3} = -\frac{1}{6\pi}\left(h^{1,1} - h^{1,2}\right) . \tag{A.48}$$

This is also the value we have used in section 4.4.

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